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AN ANALYTICAL INVESTIGATION OF THE EFFECTS OF FLOW PROPERTY FLU--ETC(U)
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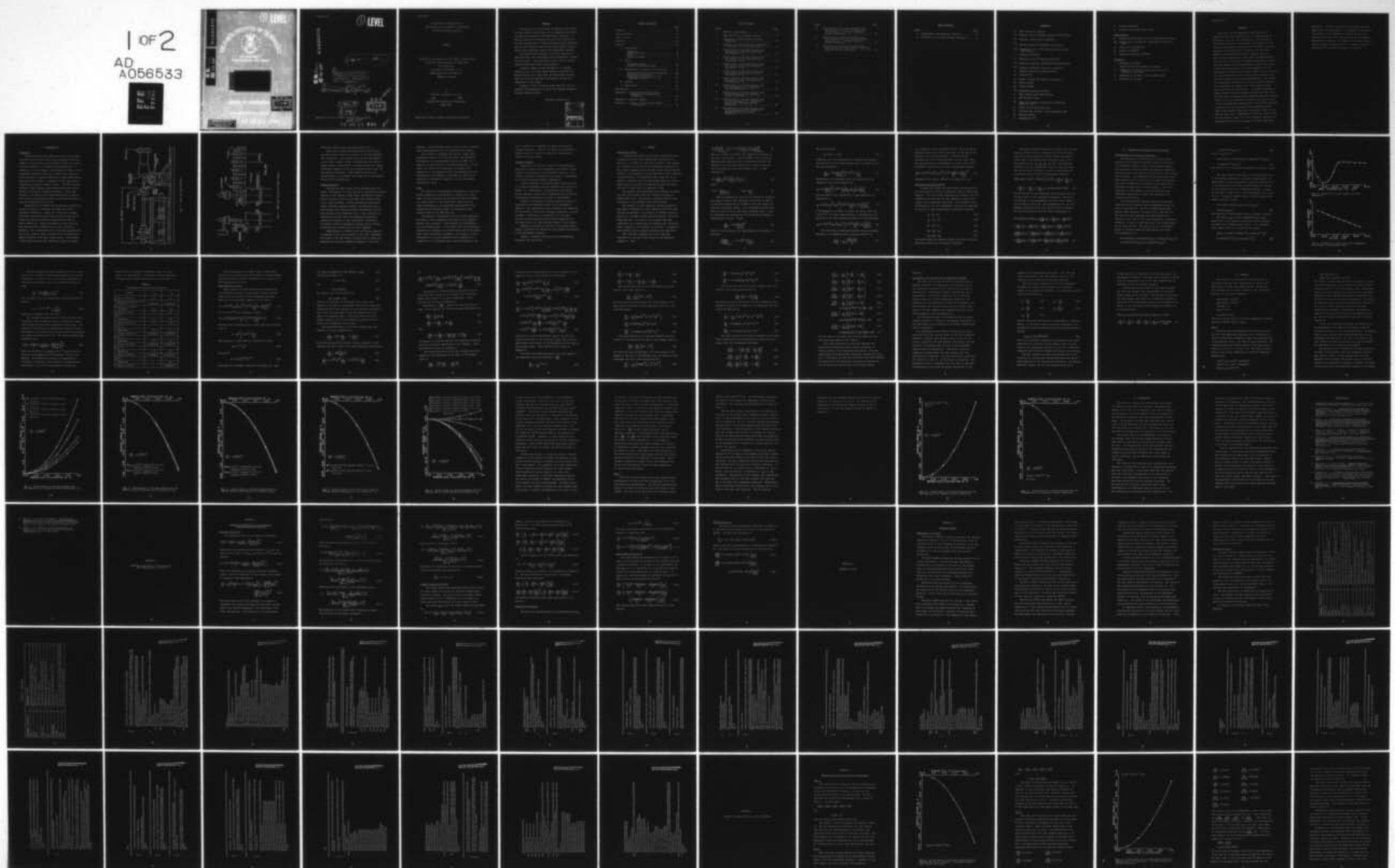

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AN ANALYTICAL INVESTIGATION OF
THE EFFECTS OF FLOW PROPERTY
FLUCTUATIONS ON THE FAY-RIDDELL
EQUATION

THESIS

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AN ANALYTICAL INVESTIGATION OF THE EFFECTS OF FLOW PROPERTY FLUCTUATIONS ON THE FAY-RIDDELL EQUATION

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by
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March 1978

Preface

The purpose of the study was to determine the effects of flow property fluctuations on the stagnation heat transfer rate as calculated by the Fay-Riddell equation. It was my intention to determine how much error resulted in the heat transfer rate if these fluctuations were neglected and the heat transfer rate was calculated using a laminar steady state expression such as the Fay-Riddell equation.

I am particularly grateful to my AFIT advisor, Dr. James Hitchcock, for his guidance and encouragement throughout the project. His instruction both in and out of the classroom made this entire study possible.

A special thanks is also extended to Dr. Sarunas Lazdinis of the Air Force Flight Dynamics Laboratory at Wright-Patterson Air Force Base, who graciously provided many long hours of assistance and support through the completion of this study.

Finally, I wish to thank my wife, Mary Ann, for her patience and acceptance of my many mood changes throughout the past eighteen months.

William H. Barnett, Jr.

Table of Contents

List of Figures

Figure		Page
14	Percent Error in the Heat Transfer Rate for Fluctuations of (a) T_s and ρ_s, and (b) T_s and ρ_w - Correlated (Baseline Initial Conditions)	86
15	Percent Error in the Heat Transfer Rate for Fluctuations of (a) T_s and P_{t2}, and (b) T_s and T_w - Correlated (Baseline Initial Conditions)	87
16	Percent Error in the Heat Transfer Rate for Fluctuations of ρ_s and T_w - Correlated (Baseline Initial Conditions)	88

List of Tables

Notations

C_j	Mass fraction of species
Cp	Specific heat at constant pressure ($ft^2/sec^2-{}^oR$)
D_j	Diffusion coefficient of species j
$D^T{}_j$	Thermal diffusion coefficient of species j
g_{lj}	Degeneracy of the ℓ^{th} electronic level of the j^{th} species
h	Enthalpy (ft^2/sec^2)
h_j	Enthalpy of the j^{th} species (ft^2/sec^2)
h_o	Reference enthalpy, $2119 \times 10^8 ft^2/sec^2$=8465 BTU/lb
$h_j{}^o$	Formation enthalpy of species j (cal/mole)
K	Thermal conductivity (BTU/hr-ft-^oR)
Le	Lewis Number
n_j	Number of atoms per molecule of species j
Nu	Nusselt Number
Pr	Prandtl Number
P_{t_2}	Stagnation pressure (lb/ft^2)
q	Heat transfer rate (BTU/ft^2-sec)
Q	$q\sqrt{R}$ (BTU/$ft^{1.5}$-sec)
Q_o	Value of Q without considering fluctuations (BTU/$ft^{1.5}$-sec)
R	Radius of the blunt body (ft)
$\bar{R}$	Universal gas constant (1.987 cal/gm-mole-^oK)
Re	Reynolds Number
T	Temperature (^oR)

u Velocity (ft/sec^2)

x Distance along body surface (ft)

Greek Symbols

ε Normalized fluctuations of five independent variables

θ_{v_j} Characteristic vibrational temperature of species j (^oK)

μ Viscosity (slugs/ft-sec)

ρ Density ($slugs/ft^3$)

ν Kinematic viscosity (ft^2/sec)

Subscript

o Reference conditions

j Identifies the j^{th} species of a mixture

ℓ Identifies the ℓ^{th} electronic level of the j^{th} species of a mixture

s Evaluated at the edge of the boundary layer

w Evaluated at the wall

Abstract

The effects of flow property fluctuations on the stagnation point heat transfer rate, as calculated by the Fay-Riddell equation, are investigated. Fluctuations in free stream temperature and density, wall temperature and density, and stagnation pressure are considered. The flow parameter perturbations are included in the Fay-Riddell equation by approximating the heat transfer rate by a Taylor Series truncated to second order and expanded about the mean values of the five independent variables. Taking the time average of this expression, the first order terms in the fluctuations drop out, and an expression for the time averaged stagnation point heat transfer rate is obtained. The second derivatives in the Taylor Series expansion are obtained analytically and checked numerically. The effects of flow property fluctuations on the Fay-Riddell equation are investigated in general for a wide range of steady flow conditions and fluctuations. A comparison between the laminar steady flow heat transfer rate and the heat transfer rate with fluctuations shows that neglecting the fluctuations in the five independent variables can lead to errors in the calculated heat transfer rate. Of the five variables considered, the fluctuations of the temperature at the edge of the boundary layer have the greatest effect on the heat transfer rate. Fluctuations of half the initial value of this variable result in a 5 to 10 percent change in the heat transfer rate over the range of initial conditions

considered. The error increases significantly with the magnitude of the fluctuations. For example, a 100 percent fluctuation in this temperature results in a 15 to 35 percent error in the heat transfer rate for the range of temperature considered. The results of the cases investigated are shown in graphical form. A computer program implementing the solution procedure is included.

I. Introduction

Background

The Re-entry Nose Tip (RENT) Test Leg of the 50 Megawatt Facility of the Air Force Flight Dynamics Laboratory (AFFDL) is being used as a ground test facility for evaluating the performance of thermal protection systems. It consists of a high pressure, high voltage arc heater, interchangeable supersonic nozzles, a linear motion model and probe carriage, an exhaust system, and a high speed Ambilog data acquisition system. A schematic of the major system components is shown in Figures 1 and 2. The RENT Test Leg produces a high enthalpy, high pressure airstream ideally suited to the study of ablation phenomena. Other high pressure, high heating rate tests dealing with transpiration cooling, and internally water cooled leading edges can also be accomplished (Ref 11:12).

Calibration of the RENT Test Leg has been attempted on several occasions by personnel from AFFDL as well as Aerotherm Corporation. During the calibration attempts, null-point calorimeters were swept across the test jet to obtain the heat flux profiles at essentially fixed arc heater conditions. Large fluctuations in heat flux have been observed for all nozzles used in all calibration attempts. The fluctuations in heat flux have appeared from run to run, probe to probe, as well as for one probe when swept through the flow from a large nozzle. Fluctuations of this type have also been observed in other arc heated

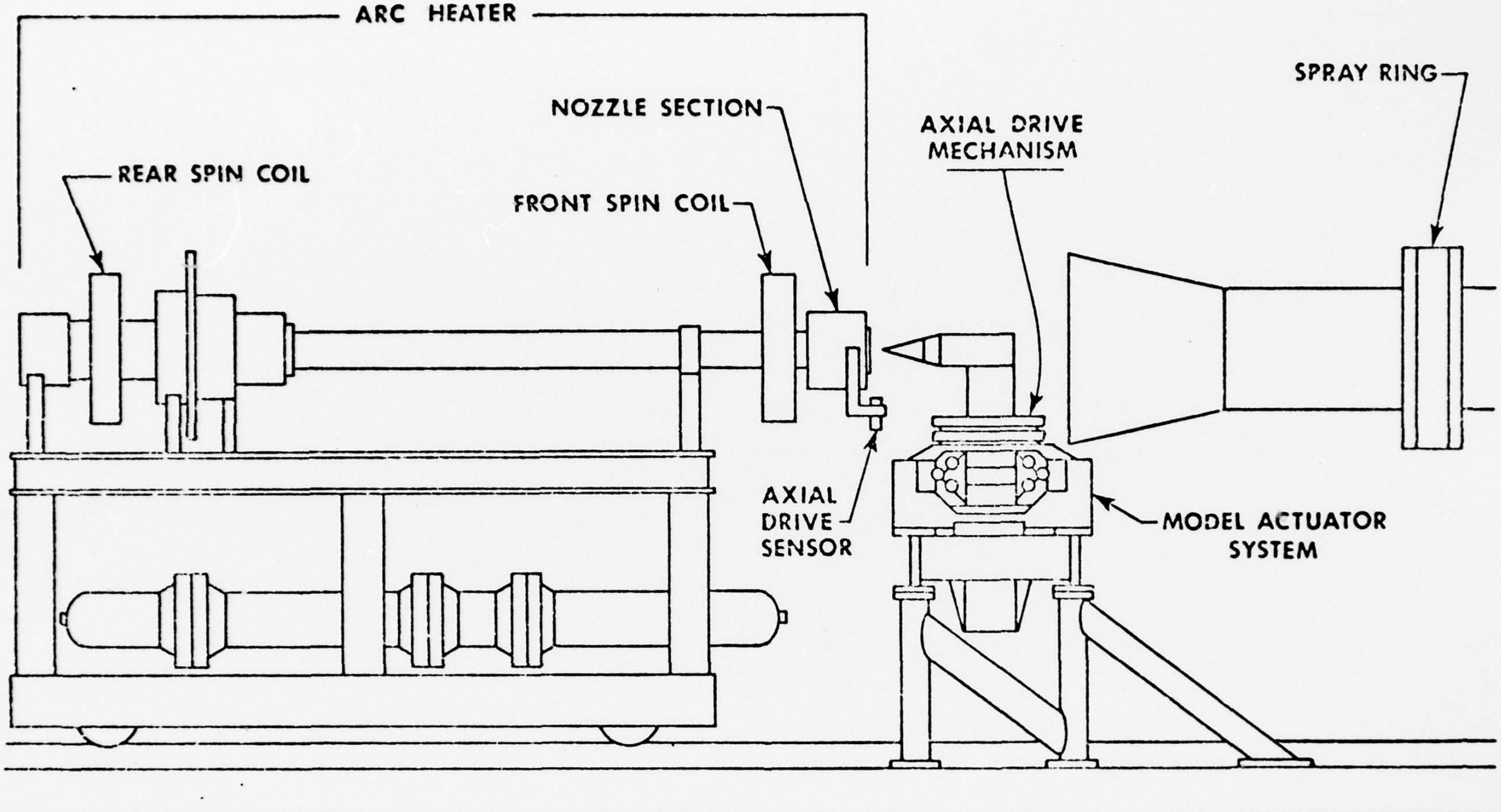

Fig. 1. RENT Test Leg Schematic

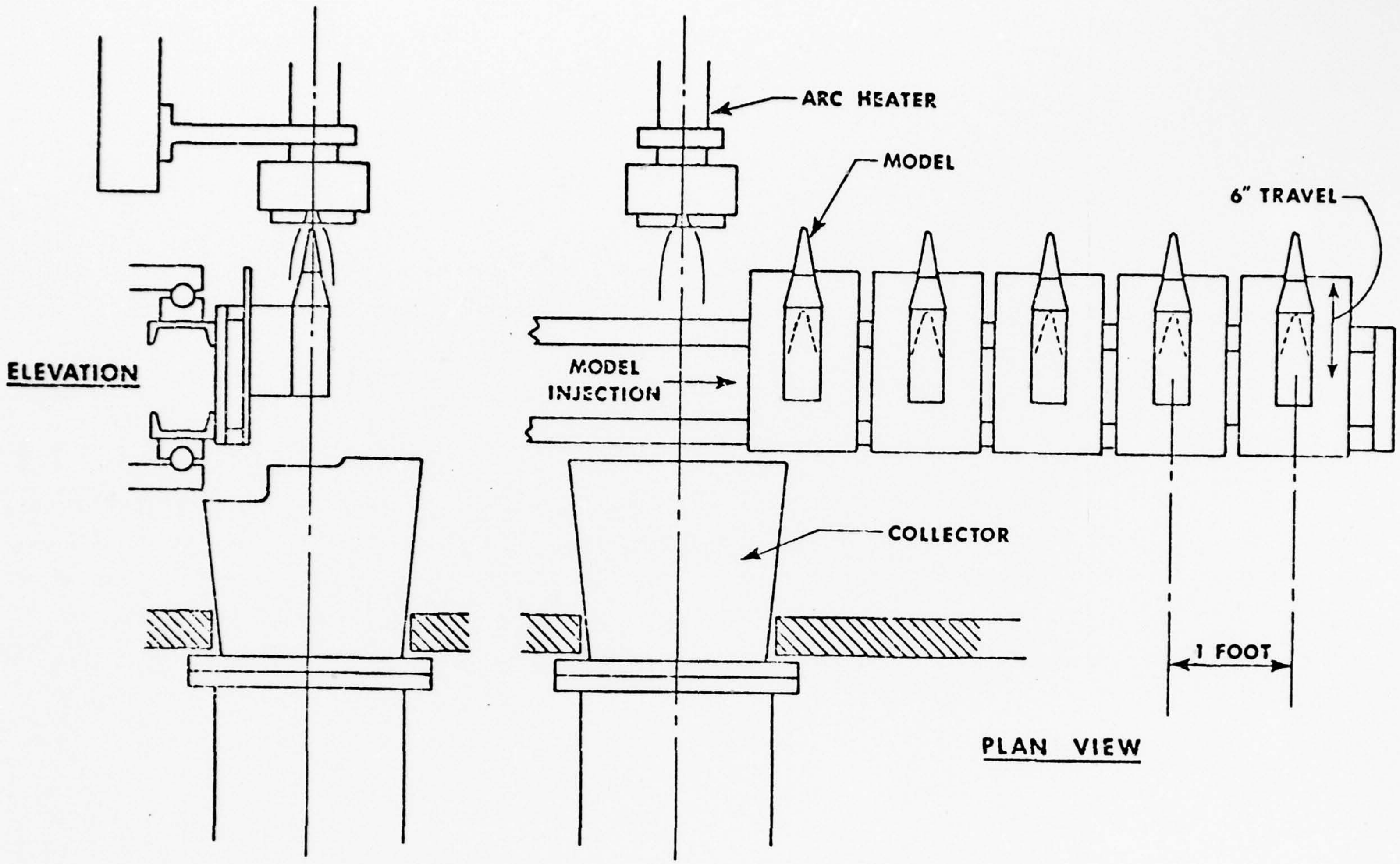

Fig. 2. RENT Test Section and Model Support

facilities used for nose tip testing (Ref 2:31).

There exist several mechanisms within the facility arc heater which could be the source of the heat transfer rate variations. Some possibilities are the fluctuations in arc heater supply voltage, fluctuations in arc heater supply current, variations in the high pressure air supply, rotational frequency of the arc attachment points, and the arc shunting frenquency. More extensive studies are required before a conclusion can be made as to the definite source of the fluctuations.

Purpose of Study

Although the exact source of the fluctuations is unknown, investigation into the effects of flow property perturbations on the laminar, steady state, Fay-Riddell equation, which is presently used to calculate heat transfer rates, is of current interest. During calibration and ablation tests, the AFFDL personnel measure the stagnation point heat transfer rate using null-point calorimeters with the slug located at the stagnation point. Also, during both calibration and test runs, the stagnation pressure is measured and appears to have significant fluctuations. An average centerline stagnation pressure and an average centerline heat transfer rate are then computed.

Interpretation of ablation nose tip testing requires a knowledge of the total enthalpy of the flow. Currently, the theory of Fay and Riddell is being used to relate the measured heat flux and stagnation pressure to the total

enthalpy. The Fay-Riddell theory is also used to evaluate other flow parameters such as temperature and density.

The validity of applying the laminar, steady state, Fay-Riddell equation to a facility which has significant fluctuations in flow parameters such as the RENT, is a question of great concern to the AFFDL engineers. In this study, the effects of fluctuations of various flow parameters on the stagnation point heat transfer rate are investigated in an attempt to determine the validity of applying the Fay-Riddell equation to a flow field where fluctuations exist.

Scope

The effects of flow parameter fluctuations on the Fay-Riddell equation are investigated in general for a wide range of flow conditions and fluctuations that could apply to a large number of test facilities. Then a more in depth investigation is accomplished using specific flow conditions and realistic ranges of fluctuations that actually exist in the RENT Test Leg.

The effects of fluctuations in free stream temperature and density, wall temperature and density, and stagnation pressure are considered. These effects are investigated for two limiting cases. In the first case, the fluctuating components are assumed to be uncorrelated, and in the second case, total correlation is assumed. For purposes of this investigation, the heat transfer rate is assumed to be insensitive to the boundary layer chemical reactions. The

air is assumed to be composed of oxygen molecules and nitrogen molecules. However, the study is performed in a general manner to allow for inclusion of dissociation effects in a later study.

Outline of Report

This report is written to describe the mathematical modelling procedure used to determine the effects of flow property fluctuations on the Fay-Riddell equation. The solution procedure described in this report is implemented in a computer program listed in Appendix B.

A brief discussion of the Fay-Riddell theory and how it is used to determine the heat transfer rate is given in Chapter II. Additionally, a discussion of the Taylor Series expansion method used to mathematically include the flow property fluctuations is given in this chapter.

Chapter III provides the actual mechanics involved in developing the mathematical model of the Fay-Riddell equation. The expressions used to calculate the parameters in the equation, such as the transport properties, are also included. Additionally, a description of the steps involved in developing the Taylor Series expansion and the coefficients for this series are shown in this chapter.

Chapter IV includes the results of the cases investigated to determine the effects of flow property fluctuations on the Fay-Riddell equation.

Chapter V contains conclusions and recommendations resulting from this study.

II. Theory

Fay-Riddell Theory

A detailed investigation of laminar stagnation point heating with dissociation effects was reported by J. A. Fay and F. R. Riddell in 1958 (Ref 4:78-85). They carried out a numerical solution of the differential equations of the laminar stagnation point boundary layer using tabulated values of the properties of high temperature air in dissociation equilibrium. The results of this study led to an equation, referred to as the Fay-Riddell equation, that is widely used to calculate the stagnation point heat transfer rate of blunt bodies in hypersonic flow.

The boundary layer equations were developed in general for high speed flight where the external flow was in a dissociated state. The effects of diffusion and of atom ionization in the boundary layer were included.

For purposes of their analysis, certain assumptions were made to establish the flow condition about the body. The gas was assumed to be a mixture of thermally perfect gases (i.e. the gas, as well as each component, obeys the ideal gas law). The flow in the boundary layer was considered to be laminar and the boundary layer thickness small compared to the curvature of the body. In addition, mass diffusion due to pressure gradients was neglected.

The local heat transfer rate to the body was determined by the sum of the conductive and diffusive transports. Thus

$$q = \left[K\left(\frac{\partial T}{\partial y}\right) \right]_{y=0} + \left[\sum \rho(h_j - h_j^{\circ}) \left\{ D_j \left(\frac{\partial c_i}{\partial y}\right) + \left(\frac{D_j^T c_i}{T}\right)\left(\frac{\partial T}{\partial y}\right) \right\} \right]_{y=0} \tag{1}$$

where the x and y directions are tangential and normal to the body, respectively. Fay and Riddell then introduced dimensionless temperature and enthalpy distributions and simplified the previous expression to obtain an equation for the stagnation point heat transfer rate. It was expressed as

$$q = \left(\frac{Nu}{\sqrt{Re}}\right) \sqrt{\rho_w \mu_w \left(\frac{\partial u_e}{\partial x}\right)_s} \left[\frac{h_s - h_w}{Pr}\right] \tag{2}$$

where

$$Nu = \frac{q x C_{P_w}}{K_w (h_s - h_w)} \quad ; \qquad Re = \frac{u_e x}{\nu_w} \tag{3}$$

Then non-dimensional forms of the continuity, momentum, and energy boundary layer equations were solved to derive a simplified expression for $Nu/\sqrt{Re}$ for both the frozen and equilibrium boundary layers. This investigation deals only with the results of the equilibrium boundary layer. First, an expression was obtained for the $Nu/\sqrt{Re}$ ratio assuming a Lewis Number of unity. It was

$$\frac{Nu}{\sqrt{Re}} = 0.67\left(\frac{\rho_s \mu_s}{\rho_w \mu_w}\right)^{0.4} \tag{4}$$

Then for other values of the Lewis Number, a correction of the form

$$\frac{Nu/\sqrt{Re}}{\left(Nu/\sqrt{Re}\right)_{Le=1}} = 1 + \left(Le^{0.52} - 1\right)\left(\frac{h_D}{h_s}\right) \tag{5}$$

was included where

$$h_D = \Sigma\, c_{is}\,(-h_j^{\circ}) \tag{6}$$

Combining these two expressions, an equation was obtained for the ratio of the Nusselt Number over the square root of the Reynolds Number.

$$\frac{Nu}{\sqrt{Re}} = 0.67\left(\frac{\rho_s \mu_s}{\rho_w \mu_w}\right)^{0.4}\left[1 + (Le^{0.52} - 1)\,\frac{h_D}{h_s}\right] \tag{7}$$

Substituting this expression into the equation for the stagnation point heat transfer rate

$$q = 0.67\left(\frac{\rho_s \mu_s}{\rho_w \mu_w}\right)^{0.4}\left[1 + (Le^{0.52} - 1)\,\frac{h_D}{h_s}\right]\frac{(h_s - h_w)}{Pr}\sqrt{\rho_w \mu_w \left(\frac{\partial u_e}{\partial x}\right)_s} \tag{8}$$

Fay and Riddell then assumed Pr=0.71 and simplified the expression to

$$q = 0.94\,(\rho_w \mu_w)^{0.1}(\rho_s \mu_s)^{0.4}\left[1 + (Le^{0.52} - 1)\,\frac{h_D}{h_s}\right](h_s - h_w)\sqrt{\left(\frac{\partial u_e}{\partial x}\right)_s} \tag{9}$$

A recommendation was made to replace the factor 0.94 with 0.76 $Pr^{-0.6}$ for other Prandtl Numbers. Incorporating that factor into the above expression resulted in the following:

$$q = 0.76\,Pr^{-0.6}(\rho_w \mu_w)^{0.1}(\rho_s \mu_s)^{0.4}\left[1 + (Le^{0.52} - 1)\,\frac{h_D}{h_s}\right](h_s - h_w)\sqrt{\left(\frac{\partial u_e}{\partial x}\right)_s} \tag{10}$$

For this study, a modified Newtonian flow was assumed; therefore, the stagnation point velocity gradient is

$$\left.\frac{\partial u_e}{\partial x}\right)_s = \frac{1}{R}\sqrt{\frac{2(P_s - P_\infty)}{\rho_s}} \tag{11}$$

for a spherical nose in hypersonic flow. The free stream pressure is much less than the pressure at the edge of the boundary layer and was neglected. Substituting the expression for the velocity gradient into the heat transfer rate equation and rearranging, the final form of the Fay-Riddell equation used for this analysis was obtained.

$$q\sqrt{R} = 0.76\, Pr^{-0.6} \left(\rho_w \mu_w\right)^{0.1} \left(\rho_s \mu_s\right)^{0.4} \left[1 + \left(Le^{0.52} - 1\right)\frac{h_D}{h_s}\right] \left(h_s - h_w\right) \left(\frac{2P_s}{\rho_s}\right)^{0.25} \tag{12}$$

Throughout this report, $q\sqrt{R}$ will be referred to by Q.

Fluctuation Modelling Theory

A method was chosen that would enable incorporation of the flow property variable fluctuations into the Fay-Riddell equation. The free stream temperature and density, the wall temperature and density, and the stagnation pressure were chosen as the independent variables. For purposes of this investigation, the five independent variables are expressed in terms of a time averaged and a fluctuating component.

$$T_s = \overline{T_s} + T_s' \tag{13}$$

$$T_w = \overline{T_w} + T_w' \tag{14}$$

$$\rho_s = \overline{\rho_s} + \rho_s' \tag{15}$$

$$\rho_w = \overline{\rho_w} + \rho_w' \tag{16}$$

$$P_{t_2} = \overline{P_{t_2}} + P_{t_2}' \tag{17}$$

The barred quantities denote average values and the primed quantities denote the fluctuating components.

Following the method introduced by Weeks (Ref 12), and used by Lazdinis (Ref 9), stagnation point heat transfer rate can be approximated by a Taylor Series truncated to second order, and expanded about the mean values of the five parameters $\overline{T}_s$, $\overline{T}_w$, $\overline{\rho}_s$, $\overline{\rho}_w$, and $\overline{P}_{t_2}$. The resulting expression for the instantaneous value of the heat transfer rate is thus a function of both the average and the fluctuating terms of the five independent variables.

$$Q(\rho_s, \rho_w, T_s, T_w, P_{t_2}) = Q_o(\overline{\rho_s}, \overline{\rho_w}, \overline{T_s}, \overline{T_w}, \overline{P_{t_2}}) + \frac{\partial Q}{\partial \rho_s}\rho_s' + \frac{\partial Q}{\partial \rho_w}\rho_w'$$

$$+ \frac{\partial Q}{\partial T_s}T_s' + \frac{\partial Q}{\partial T_w}T_w' + \frac{\partial Q}{\partial P_{t_2}}P_{t_2}' + \text{SECOND ORDER TERMS} \qquad (18)$$

The derivatives in this expression are evaluated using the average values of the independent variables. Taking the time average of this expression, the first order terms in the fluctuations drop out, and an expression for the time averaged stagnation point heat transfer rate is obtained. The resulting equation used for this investigation is

$$Q = Q_o(\overline{\rho_s}, \overline{\rho_w}, \overline{T_s}, \overline{T_w}, \overline{P_{t_2}}) + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial \rho_s^2}\langle \rho_s'^2\rangle + \frac{\partial^2 Q}{\partial \rho_w^2}\langle \rho_w'^2\rangle + \frac{\partial^2 Q}{\partial T_s^2}\langle T_s'^2\rangle\right.$$

$$+ \frac{\partial^2 Q}{\partial T_w^2}\langle T_w'^2\rangle + \frac{\partial^2 Q}{\partial P_{t_2}^2}\langle P_{t_2}'^2\rangle + 2\frac{\partial^2 Q}{\partial \rho_s \partial \rho_w}\langle \rho_s'\rho_w'\rangle + 2\frac{\partial^2 Q}{\partial \rho_s \partial T_s}\langle \rho_s' T_s'\rangle$$

$$+ 2\frac{\partial^2 Q}{\partial \rho_s \partial T_w}\langle \rho_s' T_w'\rangle + 2\frac{\partial^2 Q}{\partial \rho_s \partial P_{t_2}}\langle \rho_s' P_{t_2}'\rangle + 2\frac{\partial^2 Q}{\partial \rho_w \partial T_s}\langle \rho_w' T_s'\rangle + 2\frac{\partial^2 Q}{\partial \rho_w \partial T_w}\langle \rho_w' T_w'\rangle$$

$$\left. + 2\frac{\partial^2 Q}{\partial \rho_w \partial P_{t_2}}\langle \rho_w' P_{t_2}'\rangle + 2\frac{\partial^2 Q}{\partial T_w \partial T_s}\langle T_w' T_s'\rangle + 2\frac{\partial^2 Q}{\partial T_s \partial P_{t_2}}\langle T_s' P_{t_2}'\rangle + 2\frac{\partial^2 Q}{\partial T_w \partial P_{t_2}}\langle T_w' P_{t_2}'\rangle\right] \qquad (19)$$

III. Mathematical Formulation of the Problem

Thermodynamic and Transport Properties

The mechanics of formulating the problem involve applying the fluctuation modelling theory to the form of the Fay-Riddell equation shown in the previous chapter. This procedure involves writing the transport properties and parameters in the equation in terms of the five independent variables. The required derivatives are then determined analytically to allow for an in depth investigation into which parameters in the equation are significantly affecting the magnitude of the derivatives. Due to the complexity of the equation, the derivatives are also determined numerically to ensure the accuracy of the analytical derivatives.

In calculating heat transfer rates at high speeds, variations of the transport and thermodynamic properties must be accounted for. Curve fit Prandtl Number data, used by AFFDL personnel, is used in this study (Ref 10). The curve fit data expresses the Prandtl Number as a function of the ratio of the wall enthalpy and a reference enthalpy. If h_w/h_o is less than 0.005, the Prandtl Number is set equal to 0.77. For h_w/h_o up to 2.0, the Prandtl Number is computed using the least square curve fits.

For $0.005 \leq h_w/h_o \leq 0.1$:

$$Pr = 0.8245251 - 0.1212104 \times 10^2 (h_w/h_o) + 0.349395 \times 10^3 (h_w/h_o)^2 - 0.4212513 \times 10^4 (h_w/h_o)^3 + 2.404883 \times 10^5 (h_w/h_o)^4$$

$$-0.5307031x10^{3}(h_w/h_o)^5 \qquad (20)$$

For $0.1 \leq h_w/h_o \leq 2.0$:

$$Pr=0.788199-0.1287464(h_w/h_o)+0.238641x10^{-1}(h_w/h_o)^2$$

$$+0.102987x10^{-1}(h_w/h_o)^3 \qquad (21)$$

The variation of Pr with wall temperature is shown in Figure 3.

The Lewis Number is the ratio of the mass diffusivity to the thermal diffusivity. Like the Prandtl Number, the Lewis Number is also calculated by curve fit data (Ref 10). It is a function of the ratio of the enthalpy at the edge of the boundary layer and a reference enthalpy. If h_s/h_o is less than or equal to 0.1, the Lewis Number is set equal to 1.4. If h_s/h_o is greater than 0.1, the following least square curve fit is used:

$$Le=1.43691-0.54919(h_s/h_o)-0.0716(h_s/h_o)^2$$

$$+0.06833(h_s/h_o)^3 \qquad (22)$$

The variation of Lewis Number with T_s is shown in Figure 4.

The ratio h_D/h_s is also calculated from a least square curve fit of data as a function of the enthalpy at the edge of the boundary layer (Ref 1). The following least square curve fit is used for this ratio:

$$h_D/h_s=-0.124554+0.22780x10^{-3}h_s-0.294417x10^{-7}h_s^{\;2}$$

$$+0.184600x10^{-11}h_s^{\;3}-0.432111x10^{-16}h_s^{\;4} \qquad (23)$$

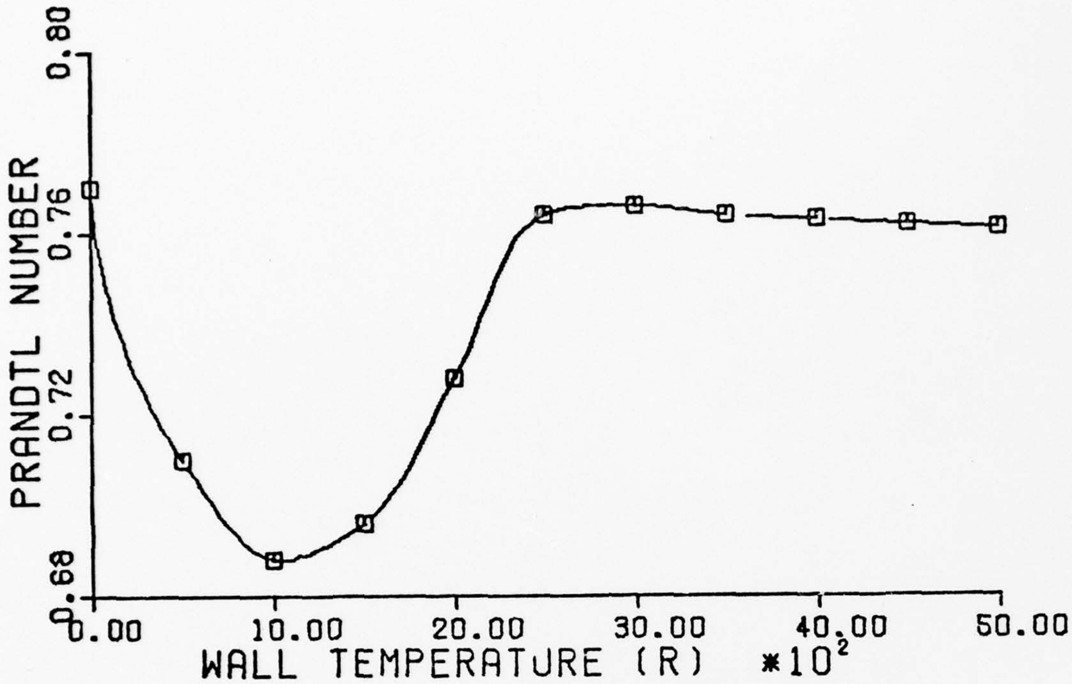

Fig. 3. Variation of Prandtl Number with Wall Temperature

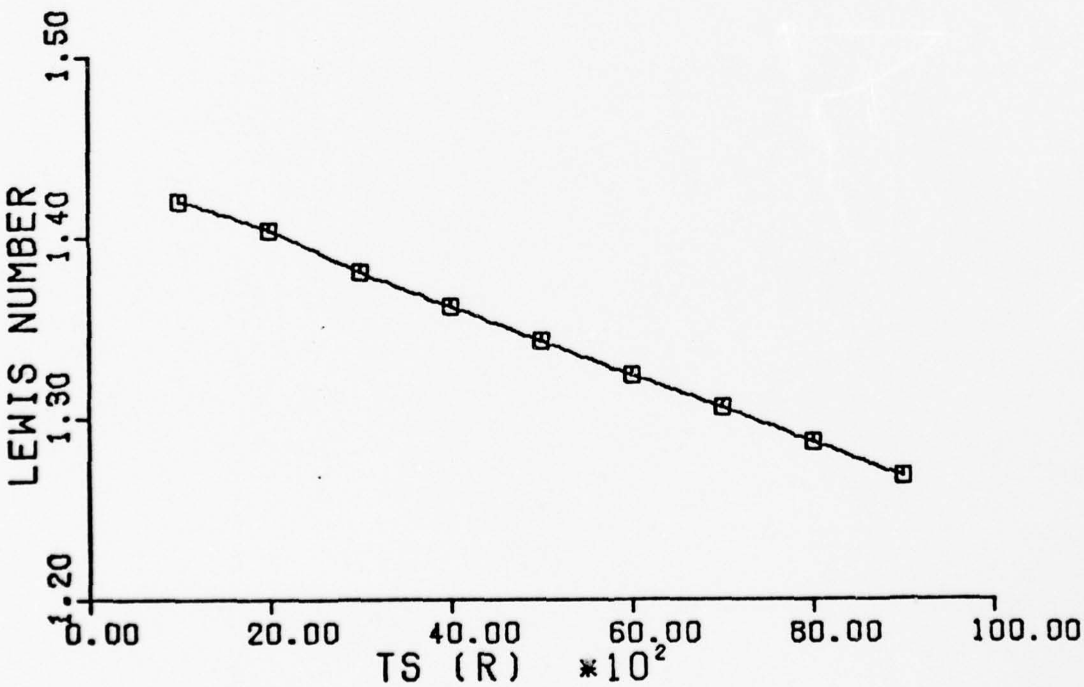

Fig. 4. Variation of Lewis Number with Temperature at the Edge of the Boundary Layer

For the temperature ranges considered in this investigation, the viscosity is essentially dependent only on the temperature. To account for the viscosity variations in the high speed boundary layer, the Sutherland Formula is used.

$$\frac{\mu}{\mu_r} = \frac{T_r + 198.6}{T + 198.6}\left(\frac{T}{T_R}\right)^{1.5} \tag{24}$$

The viscosity of air for temperatures in degrees Rankine is given by

$$\mu = 2.27 \times 10^{-8} \, \frac{T^{1.5}}{T + 198.6} \tag{25}$$

where μ is in slugs/ft-sec (Ref 5:49).

The enthalpy was calculated in a form that would allow inclusion of all the species of air, although only oxygen and nitrogen molecules were considered in this study. The enthalpy of the mixture is determined by taking the sum of the product of the species mole fractions and the corresponding species enthalpy (Ref 10:17). The species enthalpy was calculated from

$$\frac{h_j - h_j^{\circ}}{T} = \frac{5 + 2(n_j - 1)}{2} + \frac{(n_j - 1)\,\theta_{v_j}}{T\left(e^{\theta_{v_j}/T} - 1\right)} + \frac{\sum_{\ell=1}^{L_j} \varepsilon_{\ell_j}\, g_{\ell_j}\, e^{-\varepsilon_{\ell_j}/T}}{T \sum_{\ell=1}^{L_j} g_{\ell_j}\, e^{-\varepsilon_{\ell_j}/T}} \tag{26}$$

where the subscript j represents the j^{th} species and the subscript ℓ represents the ℓ^{th} electronic level of that species. This expression includes the translational, rotational, vibrational, and electronic contributions of each species. The use of this equation requires the

specification of molecular constants n_j, Θ_{v_j}, h_j^o, $g_{\ell j}$, and $\varepsilon_{\ell j}$ for each species. The values of these quantities for oxygen and nitrogen molecules are given in Table I.

Table I

Thermodynamic and Molecular Constants

Constant	N_2	O_2
Atoms Per Molecule	2.0	2.0
Characteristic Vibrational Temperature (^oK)	3.35324×10^3	2.23897×10^3
Enthalpy of Formation (cal/mole)	0.0	0.0
Electronic Levels	4.0	5.0
Level I		
Degeneracy	1.0	3.0
Energy (cal/mole)	0.0	0.0
Level II		
Degeneracy	3.0	2.0
Energy (cal/mole)	1.43685×10^5	2.26370×10^4
Level III		
Degeneracy	6.0	1.0
Energy (cal/mole)	1.70475×10^5	3.77250×10^4
Level IV		
Degeneracy	1.0	3.0
Energy (cal/mole)	1.71540×10^5	1.03198×10^5
Level V		
Degeneracy	-	3.0
Energy (cal/mole)	-	1.42390×10^5

These expressions for Prandtl Number, Lewis Number, viscosity, and enthalpy are substituted into the Fay-Riddell equation to express the heat transfer rate in terms of the five independent variables.

Analytical Derivatives

Before taking the desired derivatives analytically, the Fay-Riddell equation is rearranged to group together terms that are functions of each independent variable. Start with the Fay-Riddell equation in the form of

$$Q = 0.76\, Pr^{-0.6} (\rho_w \mu_w)^{0.1} (\rho_s \mu_s)^{0.4} \left[1 + (Le^{0.52} - 1)\frac{h_D}{h_s}\right] (h_s - h_w) \left(\frac{2 P_{t_2}}{\rho_s}\right)^{0.25} \quad (27)$$

Let $C = 0.76(2)^{0.25}$ and rearrange to obtain

$$Q = C\, Pr^{-0.6} \mu_w^{0.1} \rho_w^{0.1} \rho_s^{0.15} P_{t_2}^{0.25} \left\{ \mu_s^{0.4} \left[1 + (Le^{0.52} - 1)\frac{h_D}{h_s}\right]\right\} (h_s - h_w) \quad (28)$$

Now define the following group of terms that are a function of T_s as

$$\alpha = \mu_s^{0.4} \left[1 + (Le^{0.52} - 1)\frac{h_D}{h_s}\right] \quad (29)$$

and the group of terms that are a function of T_w as

$$\beta = Pr^{-0.6} \mu_w^{0.1} \quad (30)$$

Also define

$$\gamma = C \rho_w^{0.1} \rho_s^{0.15} P_{t_2}^{0.25} \quad (31)$$

Rewriting the Fay-Riddell equation to include α, β , and γ

$$Q = \gamma(\rho_s, \rho_w, P_{t_2})\left[\beta(T_w)\cdot h_s \cdot \alpha(T_s) - \beta(T_w)\cdot h_w \cdot \alpha(T_s)\right] \qquad (32)$$

Simplifying further, let

$$A = h_s \, \alpha(T_s) \qquad (33)$$

and

$$B = h_w \, \beta(T_w) \qquad (34)$$

The equation can then be written as

$$Q = \gamma[\beta A - \alpha B] \qquad (35)$$

where γ is a function of the density at the wall, the density at the edge of the boundary layer and the stagnation pressure; β and B are functions of only the wall temperature; and A and α are functions of only the temperature at the edge of the boundary layer. Using this form of the Fay-Riddell equation, the second derivatives required for the Taylor Series expansion can be taken.

The second derivative of the heat transfer rate with respect to the wall temperature is

$$\frac{\partial^2 Q}{\partial T_w^2} = \gamma\left[A \frac{\partial^2 \beta}{\partial T_w^2} - \alpha \frac{\partial^2 B}{\partial T_w^2}\right] \qquad (36)$$

The first and second derivatives of β are required in this equation and also in some of the other analytical derivatives.

$$\frac{\partial \beta}{\partial T_w} = \frac{\partial(P_r^{-0.6}\mu_w^{0.1})}{\partial T_w} \qquad (37)$$

$$\frac{\partial \beta}{\partial T_w} = 0.1\, P_r^{-0.6}\mu_w^{-0.9}\frac{\partial \mu_w}{\partial T_w} - 0.6\, P_r^{-1.6}\mu_w^{0.1}\frac{\partial P_r}{\partial T_w} \qquad (38)$$

and

$$\frac{\partial^2 \beta}{\partial T_w} = 0.1 P_r^{-0.6} \mu_w^{-0.9} \frac{\partial^2 \mu_w}{\partial T_w^2} - 0.09 P_r^{-0.6} \mu_w^{-1.9} \left(\frac{\partial \mu_w}{\partial T_w}\right)^2 - 0.12 P_r^{-1.6} \mu_w^{-0.9} \frac{\partial \mu}{\partial T_w} \frac{\partial P_r}{\partial T_w}$$

$$- 0.96 P_r^{-2.6} \mu_w^{0.1} \left(\frac{\partial P_r}{\partial T_w}\right)^2 - 0.6 P_r^{-1.6} \mu_w^{0.1} \frac{\partial^2 P_r}{\partial T_w^2} \qquad (39)$$

Calculation of the β derivatives requires the first and second derivatives of the viscosity and the Prandtl Number with respect to the wall temperature. These derivatives are shown in Appendix A.

Additionally, $\frac{\partial^2 Q}{\partial T_w^2}$, as well as other derivatives taken later, requires the first and second derivatives of B.

$$\frac{\partial B}{\partial T_w} = \frac{\partial}{\partial T_w}\left(h_w \beta\right) \qquad (40)$$

$$\frac{\partial B}{\partial T_w} = h_w \frac{\partial \beta}{\partial T_w} + \beta \frac{\partial h_w}{\partial T_w} \qquad (41)$$

and

$$\frac{\partial^2 B}{\partial T_w^2} = h_w \frac{\partial^2 \beta}{\partial T_w^2} + 2 \frac{\partial h_w}{\partial T_w} \frac{\partial \beta}{\partial T_w} + \beta \frac{\partial^2 h_w}{\partial T_w^2} \qquad (42)$$

In addition to the β derivatives, this expression requires the first and second derivatives of the enthalpy. These derivatives are given in Appendix A.

The second derivative of the heat transfer rate with respect to the temperature at the edge of the boundary layer is

$$\frac{\partial^2 Q}{\partial T_s^2} = \gamma \left[\beta \frac{\partial^2 A}{\partial T_s^2} - \frac{\partial^2 \alpha}{\partial T_s^2} B\right] \qquad (43)$$

The first and second derivatives of α are required in this expression and in other analytical derivatives.

$$\frac{\partial\alpha}{\partial T_s} = \frac{\partial}{\partial T_s}\left\{\mu_s^{0.4}\left[1+(Le^{0.52}-1)\frac{h_D}{h_s}\right]\right\} \tag{44}$$

$$\frac{\partial\alpha}{\partial T_s} = 0.4\,\mu_s^{-0.6}\left[1+(Le^{0.52}-1)\frac{h_D}{h_s}\right]\frac{\partial\mu_s}{\partial T_s} + \mu_s^{0.4}\left[Le^{0.52}-1\right]\frac{\partial\left(\frac{h_D}{h_s}\right)}{\partial T_s}$$

$$+\,0.52\,\mu_s^{0.4}\left(\frac{h_D}{h_s}\right)Le^{-0.48}\,\frac{\partial Le}{\partial T_s} \tag{45}$$

and

$$\frac{\partial^2\alpha}{\partial T_s^2} = -0.24\,\mu_s^{-1.6}\left[1+(Le^{0.52}-1)\frac{h_D}{h_s}\right]\left(\frac{\partial\mu_s}{\partial T_s}\right)^2 + 0.416\,\mu_s^{-0.6}Le^{-0.48}\left(\frac{h_D}{h_s}\right)\frac{\partial Le}{\partial T_s}\,\frac{\partial\mu_s}{\partial T_s}$$

$$+\,0.8\,\mu_s^{-0.6}(Le^{0.52}-1)\,\frac{\partial\left(\frac{h_D}{h_s}\right)}{\partial T_s}\,\frac{\partial\mu_s}{\partial T_s} + 0.4\,\mu_s^{-0.6}\left[1+(Le^{0.52}-1)\left(\frac{h_D}{h_s}\right)\right]\frac{\partial^2\mu_s}{\partial T_s^2}$$

$$+\,1.04\,\mu_s^{0.4}Le^{-0.48}\,\frac{\partial Le}{\partial T_s}\,\frac{\partial\left(\frac{h_D}{h_s}\right)}{\partial T_s} + 0.52\,\mu_s^{0.4}\left(\frac{h_D}{h_s}\right)Le^{-0.48}\,\frac{\partial^2 Le}{\partial T_s^2}$$

$$+\,\mu_s(Le^{0.52}-1)\,\frac{\partial^2\left(\frac{h_D}{h_s}\right)}{\partial T_s^2} - 0.2496\,\mu_s^{0.4}\left(\frac{h_D}{h_s}\right)Le^{-1.48}\left(\frac{\partial Le}{\partial T_s}\right)^2 \tag{46}$$

Calculations of the α derivatives require the first and second derivatives of the viscosity, h_D/h_s ratio, and the Lewis Number with respect to the temperature at the edge of the boundary layer. These derivatives are shown in Appendix A.

The first and second derivatives of A with respect to the temperature are also required in $\frac{\partial^2 Q}{\partial T_s^2}$.

$$\frac{\partial A}{\partial T_s} = \frac{\partial}{\partial T_s}(h_s\alpha) \tag{47}$$

$$\frac{\partial A}{\partial T_s} = h_s \frac{\partial \alpha}{\partial T_s} + \frac{\partial h_s}{\partial T_s} \alpha \tag{48}$$

$$\frac{\partial^2 A}{\partial T_s^2} = h_s \frac{\partial^2 \alpha}{\partial T_s^2} + 2 \frac{\partial h_s}{\partial T_s} \frac{\partial \alpha}{\partial T_s} + \alpha \frac{\partial^2 h_s}{\partial T_s^2} \tag{49}$$

The second derivative of the heat transfer rate with respect to the wall density is

$$\frac{\partial^2 Q}{\partial \rho_w^2} = \frac{\partial^2 \gamma}{\partial \rho_w^2} \left[\beta \cdot A - \alpha \cdot B \right] \tag{50}$$

The first and second derivatives of γ with respect to the wall density are needed in this expression and other cross term derivatives.

$$\frac{\partial \gamma}{\partial \rho_w} = \frac{\partial}{\partial \rho_w} \left[0.76 (2)^{0.25} \rho_w^{0.1} \rho_s^{0.15} P_{t_2}^{0.25} \right] \tag{51}$$

$$\frac{\partial \gamma}{\partial \rho_w} = 0.090736 \, \rho_s^{0.15} P_{t_2}^{0.25} \rho_w^{-0.9} \tag{52}$$

$$\frac{\partial^2 \gamma}{\partial \rho_w^2} = -0.081663 \, \rho_s^{0.15} P_{t_2}^{0.25} \rho_w^{-1.9} \tag{53}$$

The second derivative of the heat transfer rate with respect to the density at the edge of the boundary layer is

$$\frac{\partial^2 Q}{\partial \rho_s^2} = \frac{\partial^2 \gamma}{\partial \rho_s^2} \left[\beta A - \alpha B \right] \tag{54}$$

The first and second derivatives of γ with respect to the density at the edge of the boundary layer are needed in this expression and other cross term derivatives.

$$\frac{\partial \gamma}{\partial \rho_s} = \frac{\partial}{\partial \rho_s} \left[(0.76)(2)^{0.25} \rho_w^{0.1} \rho_s^{0.15} P_{t_2}^{0.25} \right] \tag{55}$$

$$\frac{\partial \gamma}{\partial \rho_s} = 0.136105\ \rho_w^{0.1}\, P_{t_2}^{0.25}\, \rho_s^{-0.85} \tag{56}$$

$$\frac{\partial^2 \gamma}{\partial \rho_s^2} = -0.115689\ \rho_w^{0.1}\, P_{t_2}^{0.25}\, \rho_s^{-1.85} \tag{57}$$

The second derivative of the heat transfer rate with respect to the stagnation pressure is

$$\frac{\partial^2 Q}{\partial P_{t_2}^2} = \left[\beta A - \alpha B\right] \frac{\partial^2 \gamma}{\partial P_{t_2}^2} \tag{58}$$

The first and second derivatives of γ with respect to the stagnation pressure are needed in this expression and other cross term derivatives.

$$\frac{\partial \gamma}{\partial P_{t_2}} = \frac{\partial}{\partial P_{t_2}} \left[0.76\,(2)^{0.25}\, \rho_w^{0.1}\, \rho_s^{0.15}\, P_{t_2}^{0.25}\right] \tag{59}$$

$$\frac{\partial \gamma}{\partial P_{t_2}} = 0.226841\ \rho_w^{0.1}\, \rho_s^{0.15}\, P_{t_2}^{-0.75} \tag{60}$$

$$\frac{\partial^2 \gamma}{\partial P_{t_2}^2} = -0.1701309\ \rho_w^{0.1}\, \rho_s^{0.15}\, P_{t_2}^{-1.75} \tag{61}$$

The other analytical derivatives required in the Taylor Series expansion are the derivatives involving the cross terms of the five independent variables.

$$\frac{\partial^2 Q}{\partial T_w \partial T_s} = \gamma \left[\frac{\partial A}{\partial T_s}\frac{\partial \beta}{\partial T_w} - \frac{\partial \alpha}{\partial T_s}\frac{\partial B}{\partial T_w}\right] \tag{62}$$

$$\frac{\partial^2 Q}{\partial T_w \partial \rho_w} = \frac{\partial \gamma}{\partial \rho_w}\left[A \frac{\partial \beta}{\partial T_w} - \alpha \frac{\partial B}{\partial T_w}\right] \tag{63}$$

$$\frac{\partial^2 Q}{\partial T_w \partial \rho_s} = \frac{\partial \gamma}{\partial \rho_s}\left[A \frac{\partial \beta}{\partial T_w} - \alpha \frac{\partial B}{\partial T_w}\right] \tag{64}$$

$$\frac{\partial^2 Q}{\partial T_w \partial P_{t_2}} = \frac{\partial \gamma}{\partial P_{t_2}} \left[A \frac{\partial \beta}{\partial T_w} - \alpha \frac{\partial B}{\partial T_w} \right] \tag{65}$$

$$\frac{\partial^2 Q}{\partial T_s \partial \rho_w} = \frac{\partial \gamma}{\partial \rho_w} \left[\frac{\partial A}{\partial T_s} \beta - \frac{\partial \alpha}{\partial T_s} B \right] \tag{66}$$

$$\frac{\partial^2 Q}{\partial T_s \partial \rho_s} = \frac{\partial \gamma}{\partial \rho_s} \left[\frac{\partial A}{\partial T_s} \beta - \frac{\partial \alpha}{\partial T_s} B \right] \tag{67}$$

$$\frac{\partial^2 Q}{\partial T_s \partial P_{t_2}} = \frac{\partial \gamma}{\partial P_{t_2}} \left[\frac{\partial A}{\partial T_s} \beta - \frac{\partial \alpha}{\partial T_s} B \right] \tag{68}$$

$$\frac{\partial^2 Q}{\partial \rho_w \partial \rho_s} = \frac{\partial}{\partial \rho_s} \left[\frac{\partial \gamma}{\partial \rho_w} (\beta \cdot A - \alpha B) \right] \tag{69}$$

$$= 0.013610 \, \rho_s^{-0.85} P_{t_2}^{0.25} \rho_w^{-0.9} (\beta A - \alpha B) \tag{70}$$

$$\frac{\partial^2 Q}{\partial \rho_w \partial P_{t_2}} = \frac{\partial}{\partial P_{t_2}} \left[\frac{\partial \gamma}{\partial \rho_w} (\beta \cdot A - \alpha B) \right] \tag{71}$$

$$= 0.0226841 \, \rho_s^{0.15} P_{t_2}^{-0.75} \rho_w^{-0.9} (\beta A - \alpha B) \tag{72}$$

$$\frac{\partial^2 Q}{\partial \rho_s \partial P_{t_2}} = \frac{\partial}{\partial P_{t_2}} \left[\frac{\partial \gamma}{\partial \rho_s} (\beta A - \alpha B) \right] \tag{73}$$

$$= 0.0340262 \, \rho_w^{0.1} P_{t_2}^{-0.75} \rho_s^{-0.85} (\beta A - \alpha B) \tag{74}$$

All the terms on the right side of these equations have been previously shown in this chapter.

The second derivatives of the heat transfer rate mentioned previously were also checked numerically to ensure accuracy of the analytical derivatives. This was accomplished by using central difference formulas (Ref 7:21-22). A difference of less than one percent was noted between the magnitude of the analytical and numerical derivatives for the second derivatives used in the Taylor Series

expansion.

Development and Structure of the Mathematical Model

The model is developed so that for different sets of initial conditions, the magnitude of the fluctuations of the independent variables can be varied and all possible combinations of independent variable fluctuations can be investigated. The initial conditions are the assigned average values of the five independent variables. The effects of the fluctuations are determined by comparing the heat transfer rate computed using only the initial conditions to the heat transfer rate computed using the Taylor Series expansion shown in Equation 19.

The fluctuation model is incorporated into a Fortran computer program. A listing of the program, description of the subroutines, and a listing of the Fortran variables are given in Appendix B.

The thermodynamic and transport properties are first calculated using the initial values of the five independent variables. Knowing these properties for a given set of initial conditions, the heat transfer rate without fluctuations can be computed using the form of the Fay-Riddell equation shown in Equation 12. This value is used for comparison purposes after the heat transfer rate with fluctuations is computed. Also from the initial conditions the values of α , β , γ , A, and B can be calculated. The expressions given in Appendix A are then used to calculate the magnitude of the first and second derivatives of the

transport and thermodynamic properties. The first and second derivatives of the heat transfer rate are then computed for the given set of initial conditions.

The fluctuation components of the five independent variables are normalized with respect to the initial values of the respective variables. The normalized fluctuations are represented by

$$\varepsilon_1 = \frac{\rho_s'}{\rho_s} \qquad (75) \qquad\qquad \varepsilon_4 = \frac{T_w'}{T_w} \qquad (78)$$

$$\varepsilon_2 = \frac{\rho_w'}{\rho_w} \qquad (76) \qquad\qquad \varepsilon_5 = \frac{P_{t_2}'}{P_{t_2}} \qquad (79)$$

$$\varepsilon_3 = \frac{T_s'}{T_s} \qquad (77)$$

and the cross term derivatives are normalized in a similar manner. In the case of perfect positive correlation, the following relationship between all fluctuating variables is assumed

$$\langle \varepsilon_i \varepsilon_j \rangle = \left(\langle \varepsilon_i^2 \rangle\right)^{\frac{1}{2}} \left(\langle \varepsilon_j^2 \rangle\right)^{\frac{1}{2}} \qquad (80)$$

Since the fluctuation components are normalized, the second derivatives in the Taylor Series expansion must be multiplied by the normalizing quantity to allow for a consistent comparison with the heat transfer rate without fluctuations.

The heat transfer rate with fluctuations can now be calculated by including the second derivatives and the desired magnitude of the fluctuations considered. The percent difference between the two heat transfer rates can be

plotted against the magnitude of the fluctuations of the independent variables to show what effect different combinations of fluctuations have on the heat transfer rate as calculated by the Fay-Riddell equation.

A check was also performed for each case to ensure that the heat transfer rate approximated by the Taylor Series expansion was accurate. This was done by computing the heat transfer rate with the fluctuations included in the value of the independent variables, and comparing this result to the full Taylor Series expansion including the first derivatives.

$$Q(\rho_s+\rho_s', \rho_w+\rho_w', T_s+T_s', T_w+T_w', P_{t_2}+P_{t_2}') = Q_o(\rho_s, \rho_w, T_s, T_w, P_{t_2})$$

$$+ \frac{\partial Q}{\partial \rho_s}\rho_s' + \frac{\partial Q}{\partial \rho_w}\rho_w' + \frac{\partial Q}{\partial T_s}T_s' + \frac{\partial Q}{\partial T_w}T_w' + \frac{\partial Q}{\partial P_{t_2}}P_{t_2}' + \text{SECOND ORDER TERMS} \qquad (81)$$

IV. Results

The effects of flow parameter fluctuations on the Fay-Riddell equation were investigated in general for a wide range of steady flow conditions and fluctuations. The following set of initial conditions was chosen as a baseline for all cases considered:

ρ_s=0.005199 slugs/ft^3

ρ_w=0.003622 slugs/ft^3

T_s=7908.0 ^oR

T_w=1170.0 ^oR

P_{t_2}=7597.0 lb/ft^2

These are typical values of the five independent variables obtained from RENT Facility data.

Case I

The first case investigated involved varying each of the five flow parameters separately over a wide range of initial values while keeping the other four parameters set to the baseline values. For each of these variations, the changing independent variable was also fluctuated over a range of ϵ =0.2, 0.4, 0.6, 0.8, 1.0. The range of variations of the initial conditions for the five independent variables was

$1\text{x}10^{-6} \leq \rho_s \leq 1\text{x}10^{-2}$ (slugs/ft^3)

$1\text{x}10^{-5} \leq \rho_w \leq 1\text{x}10^{-1}$ (slugs/ft^3)

$4000 \leq T_s \leq 10{,}000$ (^oR)

$1000 \leq T_W \leq 4000$ (^oR)

$20000 \leq P_{t_2} \leq 100{,}000$ (lb/ft^2)

The results of these cases are shown in Figures 5 through 9 with the magnitude of the fluctuations plotted versus the percent error in the heat transfer rate. The percent error in the heat transfer rate is the percent difference between the heat transfer rate without fluctuations (Q_o) and the heat transfer rate with fluctuations. The value of Q_o varies each time the initial conditions are changed so it is given in the figures for each set of initial conditions investigated. Additionally, the parabolic equations of the lines are shown on each figure, as well as the coefficients determined for each set of initial conditions.

Of the five variables fluctuated, the variation of the temperature at the edge of the boundary layer had the greatest effect on the heat transfer rate. The percent error in heat transfer rate increased positively as the temperature was decreased. For $\varepsilon=0.6$, the error in the heat transfer rate was 5.3 percent of Q_o for $T_s=10{,}000^oR$ and increased to 12.5 percent for $T_s=4000^oR$.

The variations of the density at the wall, density at the edge of the boundary layer, and stagnation pressure gave similar results. As the initial values of these variables were increased, the percent error in the heat transfer rate decreased in magnitude. However, the change in the percent error was negligible compared to the change

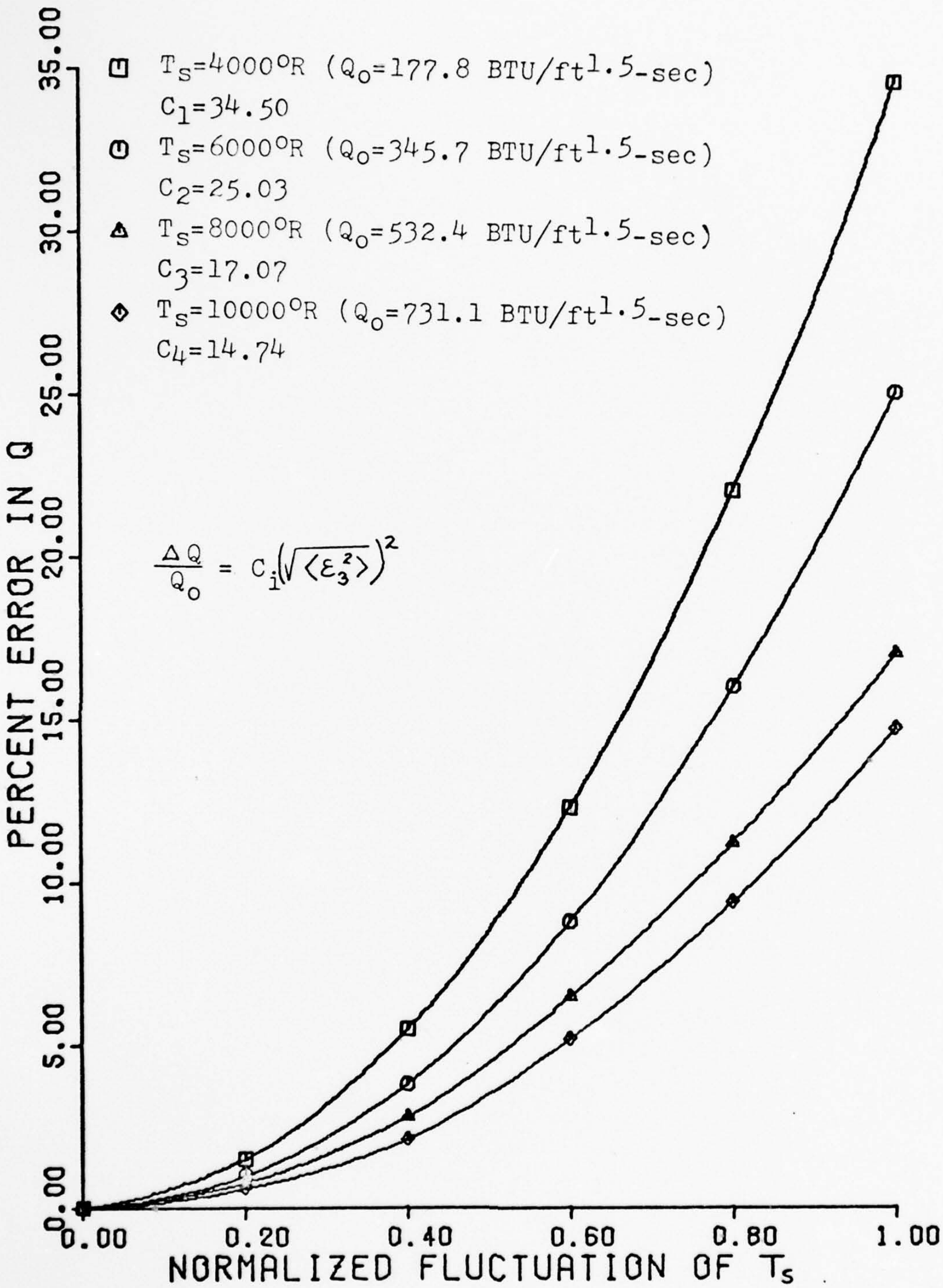

Fig. 5. Percent Error in the Heat Transfer Rate for Various Initial Conditions and Fluctuations of T_s

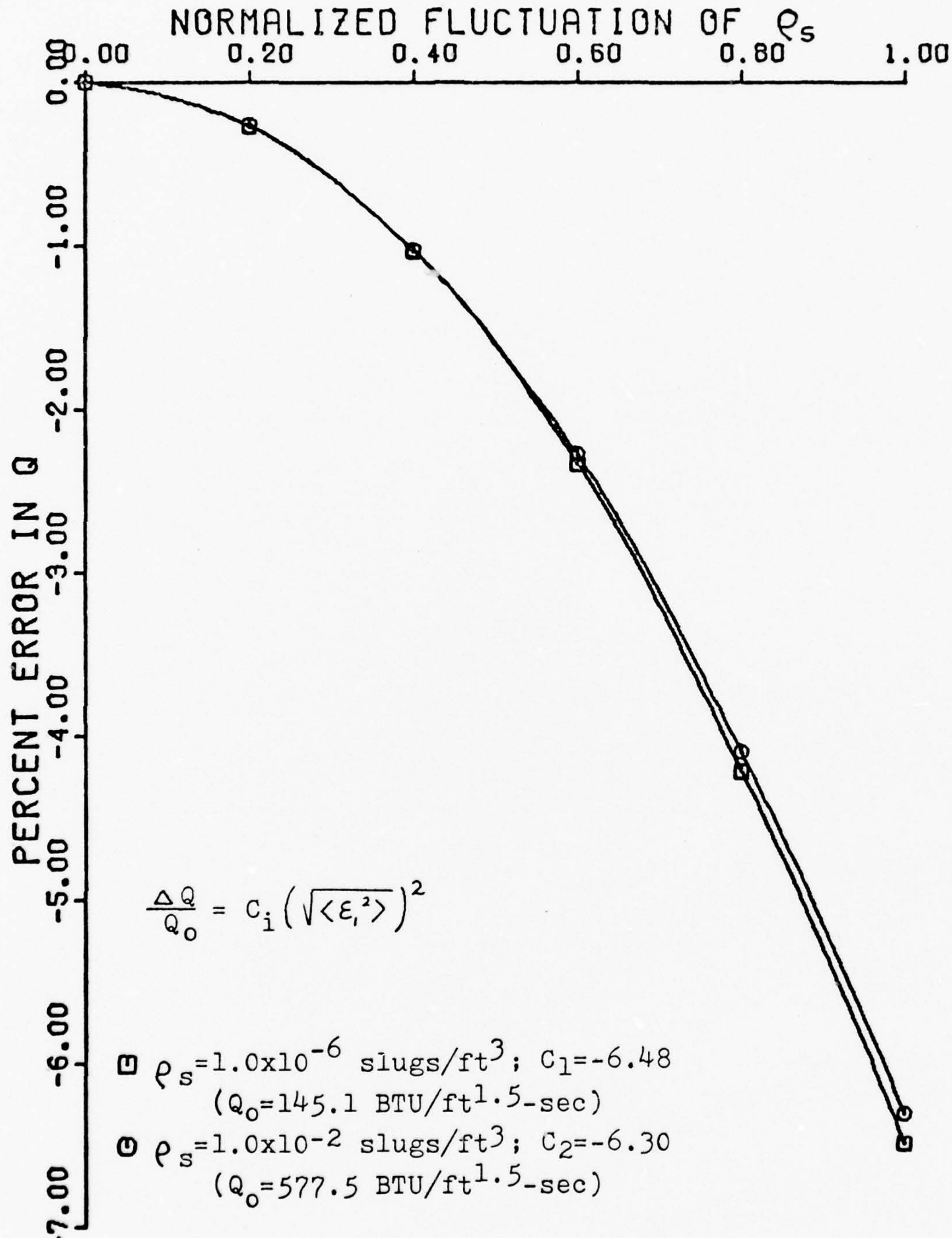

Fig. 6. Percent Error in the Heat Transfer Rate for Various Initial Conditions and Fluctuations of ρ_s

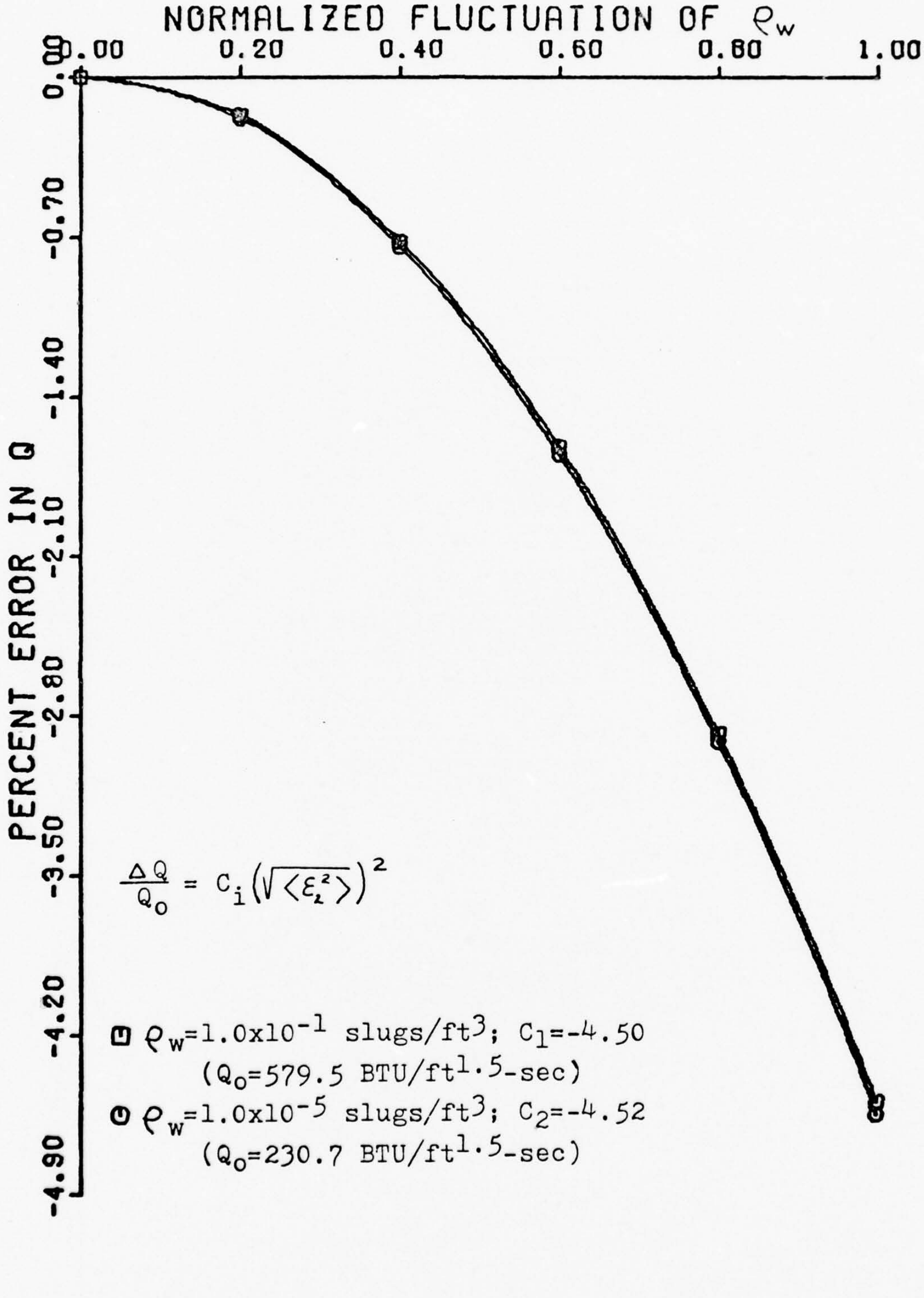

Fig. 7. Percent Error in the Heat Transfer Rate for Various Initial Conditions and Fluctuations of ρ_w

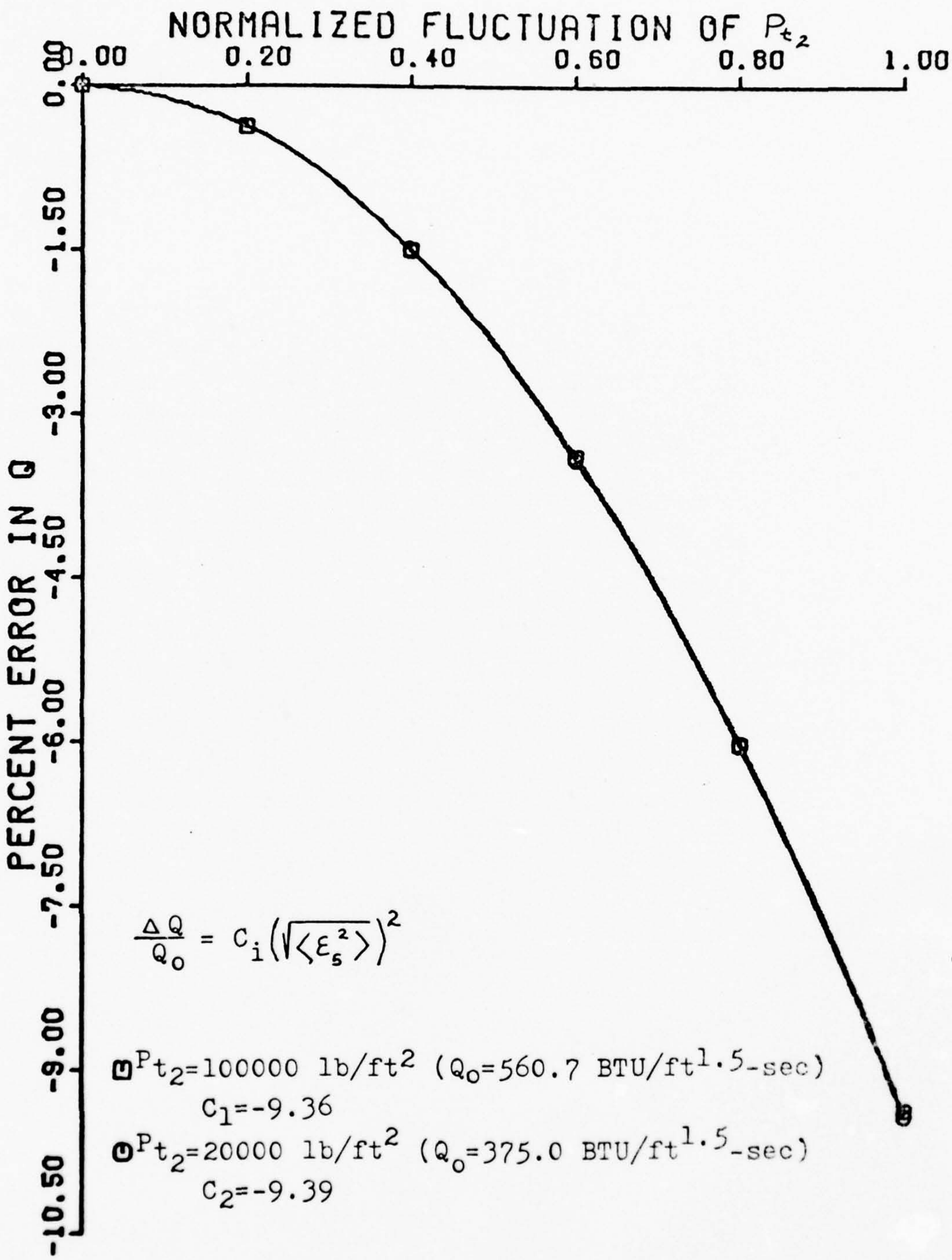

Fig. 8. Percent Error in the Heat Transfer Rate for Various Initial Conditions and Fluctuations of P_{t_2}

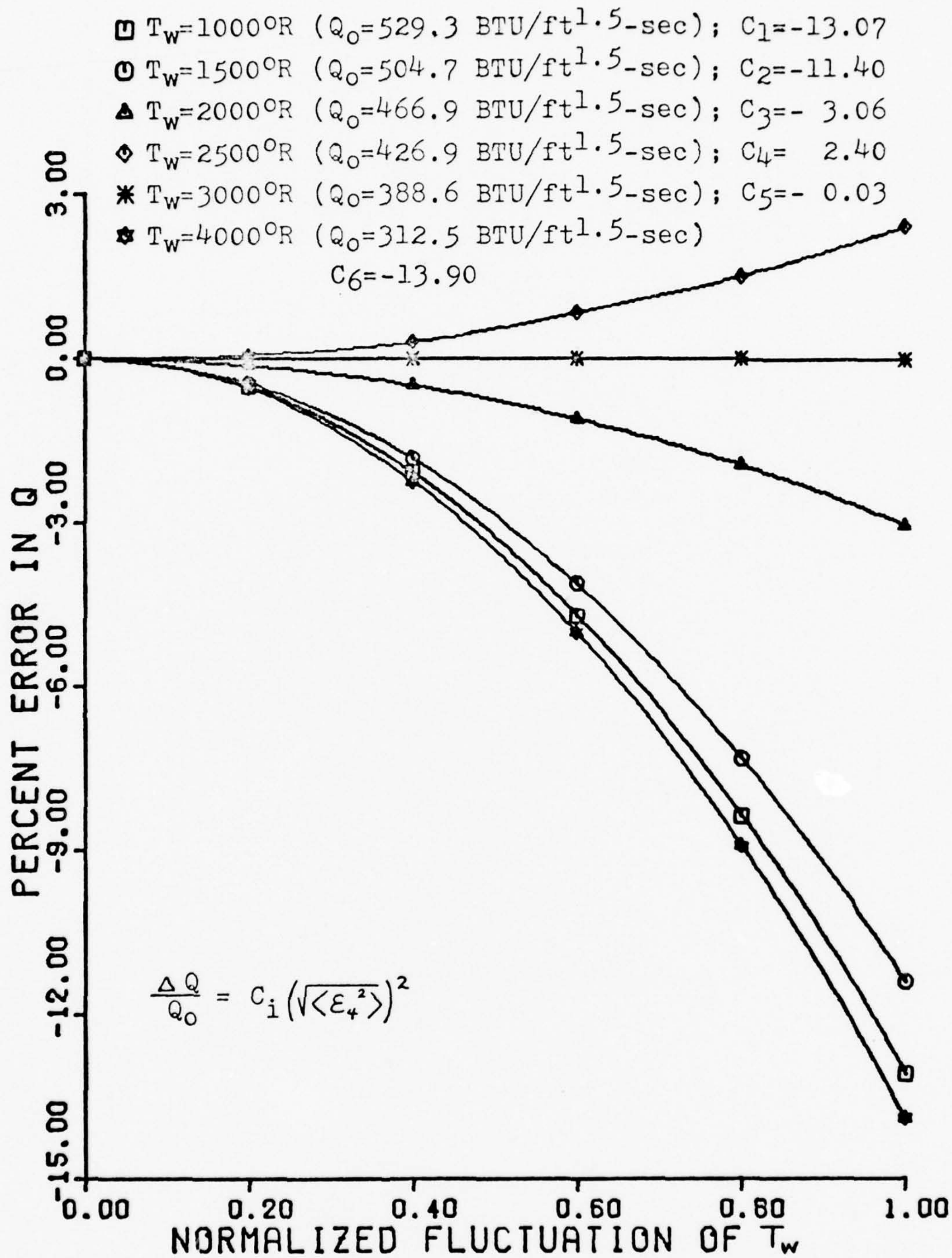

Fig. 9. Percent Error in the Heat Transfer Rate for Various Initial Conditions and Fluctuations in T_w

in the percent error for variations in the temperature initial conditions. For ε=0.6, the error in the heat transfer rate was -2.3 percent of Q_o for ρ_s=1.0x10^{-6} slugs/ft^3 and decreased to -2.2 percent for ρ_s=1.0x10^{-2} slugs/ft^3. The effect of wall density variations was slightly less than the effect of variations of the density at the edge of the boundary layer. Also, variation of the initial stagnation pressure had very little effect on the magnitude of the error in the heat transfer rate. For ε=0.6, the error in the heat transfer rate was -3.4 percent for P_{t_2}=20,000 lb/ft^2 and decreased to -3.2 percent for P_{t_2}=100,000 lb/ft^2. Figures 6, 7, and 8 show only the upper and lower limits of the ranges for initial conditions considered. Other values within each range were investigated and all curves lie between the two curves shown on each graph.

As seen from Figure 9, the initial value of the wall temperature had an effect on whether fluctuations of this variable would increase or decrease the heat transfer rate and to what extent. The variation of the wall temperature initial condition did not influence the heat transfer rate monotonically as was shown for the other four independent variables. As the wall temperature baseline was varied from 1000^{o}R to 2000^{o}R, the magnitude of the heat transfer rate error was decreased. Throughout this range, the error tended to decrease the heat transfer rate. For T_w equal to 2500^{o}R, the magnitude of the error is only

2.3 percent of Q_o but now increases the heat transfer rate. For T_w equal to $3000^{\circ}R$, the magnitude of the error is very small but again lessens the heat transfer rate. As the wall temperature baseline was varied from $3000^{\circ}R$ to $4000^{\circ}R$, the magnitude of the heat transfer rate error was increased and tended to decrease the heat transfer rate. The second derivative of the heat transfer rate with respect to the wall temperature has been shown to be a function of the first and second derivatives of β and B. The erratic behavior of $\frac{\partial^2 Q}{\partial T_w^2}$ between $2000^{\circ}R$ and $4000^{\circ}R$ can be partially explained by investigating these two derivatives. Both $\frac{\partial^2 \beta}{\partial T_w^2}$ and $\frac{\partial^2 B}{\partial T_w^2}$ are functions of the first and second derivatives of the Prandtl Number. In the range of temperatures mentioned, the Prandtl Number as a function of temperature reached a maximum value and also contained an inflection point. Therefore, the first derivative of the Prandtl Number had a sign change, and both derivatives varied significantly in magnitude in this range. These changes, coupled with changes in the other parameters in the β and B derivatives, caused the wall temperature to vary as shown previously.

Case 2

This case involved fixing the initial conditions and investigating the effects of the fluctuations of the five flow parameters independently. The initial conditions were fixed as the baseline conditions given previously in this chapter. The value of Q_o using this set of initial condi-

tions is 523.5 BTU/ft$^{1.5}$-sec. All fluctuation terms were set equal to zero except the independent variable being fluctuated. This variable was fluctuated over a range of $0 \leq \varepsilon \leq 1$.

For the entire range, fluctuations of the density at the edge of the boundary layer decreased the heat transfer rate. It was decreased by -0.3 percent for ε_1=0.2 and up to -6.4 percent for ε_1=1.0. The wall density fluctuations had less effect on the heat transfer rate than any of the other variables. The fluctuations of this variable decreased the heat transfer rate by only -4.5 percent for ε_2=1.0. Like the two densities previously discussed, the stagnation pressure fluctuations also decreased the heat transfer rate. It was decreased by -0.3 percent for ε_5=0.2 and up to -9.4 percent for ε_5=1.0.

Fluctuations in the temperature at the wall and the temperature at the edge of the boundary layer had a greater effect on the heat transfer rate than did the other three variables. The wall temperature fluctuations decreased the heat transfer rate for this set of initial conditions. It was decreased by -0.5 percent for a fluctuation of ε_4=0.2 and by -12.8 percent when a fluctuation of ε_4=1.0 was considered. Fluctuations of the boundary layer temperature had a greater effect on the heat transfer rate than did any of the other four independent variables. Additionally, these fluctuations increased the heat transfer rate in contrast to the other four variables. The fluctuations

increased the heat transfer rate by 0.7 percent for $\varepsilon_3=0.2$ and up to 17.5 percent for $\varepsilon_3=1.0$. The results of the temperature fluctuations are shown in Figures 10 and 11. The results of three other specific cases are shown in Appendix C.

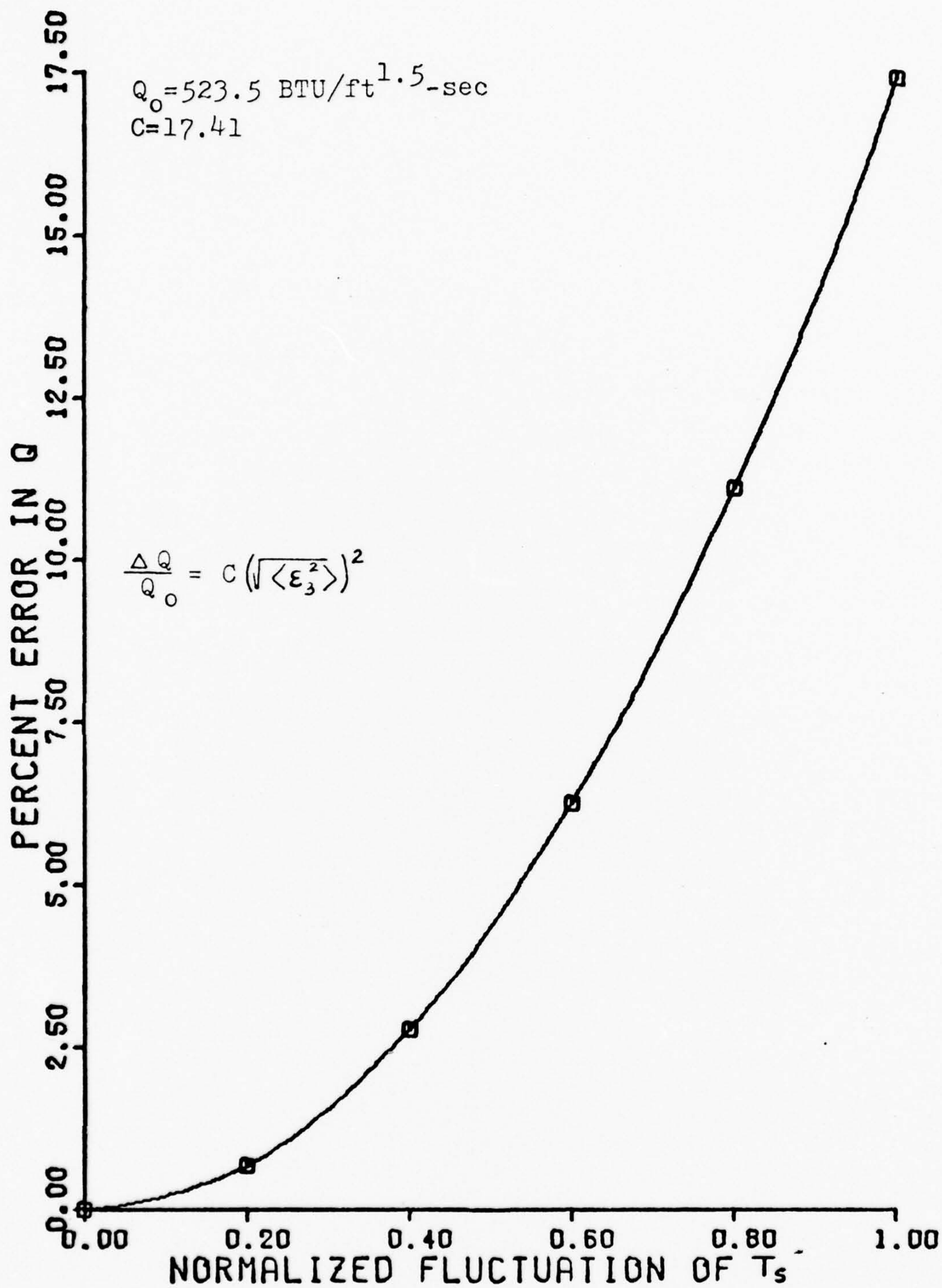

Fig. 10. Percent Error in the Heat Transfer Rate for Fluctuations of T_s (Baseline Initial Conditions)

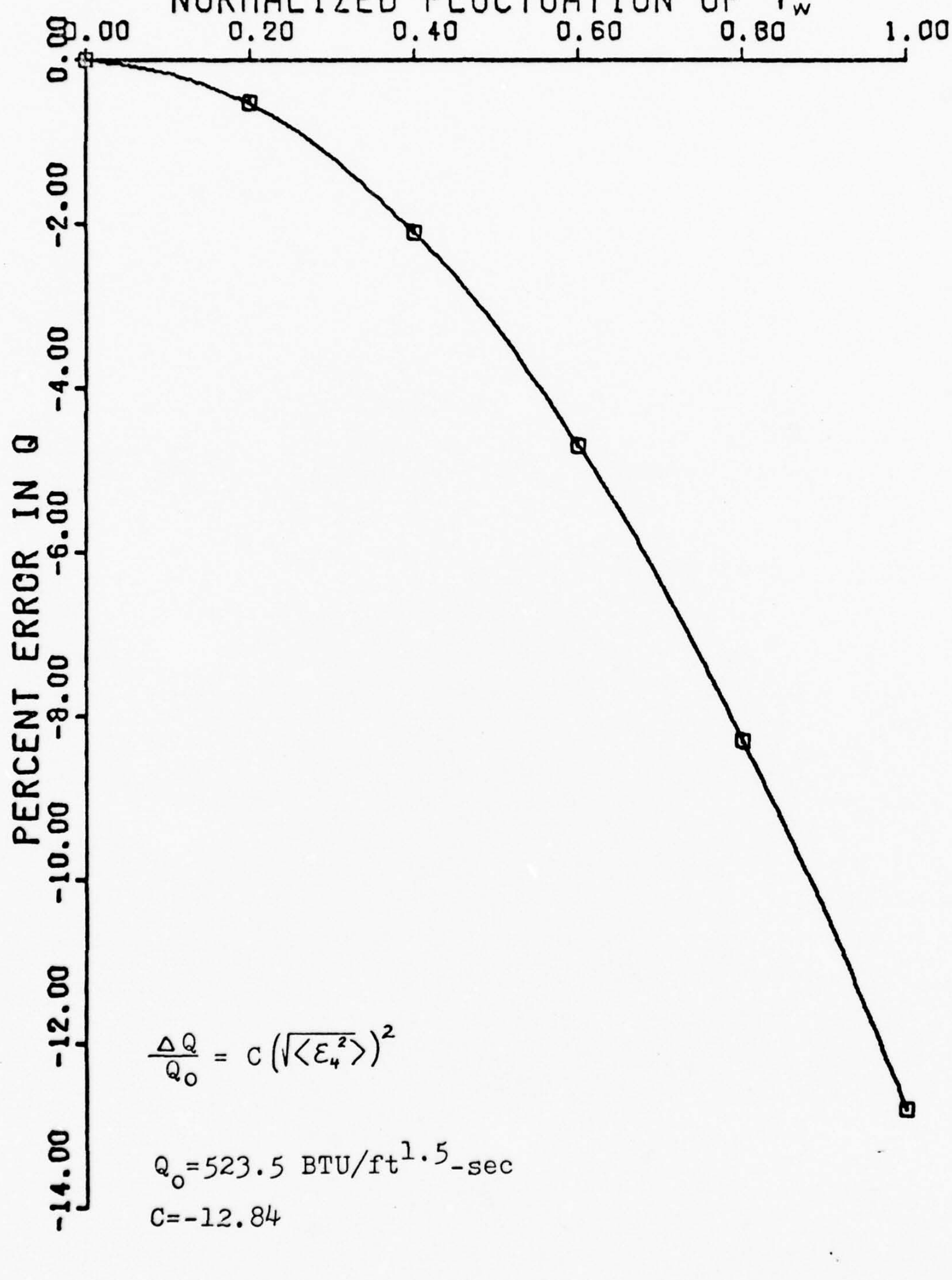

Fig. 11. Percent Error in the Heat Transfer Rate for Fluctuations of T_W (Baseline Initial Conditions)

V. Conclusions

The effect of separate and simultaneous fluctuations of temperature and density at the wall, temperature and density at the edge of the boundary layer, and the stagnation pressure, on the heat transfer rate as calculated by the Fay-Riddell equation, have been theoretically investigated. The analysis has shown that neglecting the fluctuations of these variables can lead to substantial errors in the calculated heat transfer rate. The amount of the error was found to increase with the magnitude of the fluctuations.

The initial value of the temperature at the edge of the boundary layer and the wall temperature had a significant effect on the amount of variation of the heat transfer rate. The percent error of the heat transfer rate was decreased as the initial values of both densities and the stagnation pressure were increased over a wide range of initial conditions, but the magnitude of the change was very small.

The trends in the calculated heat transfer rate, as a function of fluctuations in each of the independent variables, have been established. It has been shown that the greatest error was produced by fluctuations in the temperature at the edge of the boundary layer when all five variables were uncorrelated and only one variable fluctuated. For the baseline set of initial conditions used in this investigation, this temperature was the only one of the five variables that increased the heat transfer rate. For

the case of uncorrelation, with all variables allowed to fluctuate simultaneously, the combined negative effect of the two densities, the stagnation pressure, and the wall temperature tended to dominate the positive effect of the temperature at the edge of the boundary layer resulting in a decrease of the heat transfer rate. Again, the amount of decrease was dependent upon the magnitude of the fluctuations. With perfect positive correlation and all variables allowed to fluctuate simultaneously, it was found that the heat transfer rate increased for all cases investigated. It was pointed out that certain cross terms have a much greater effect on the heat transfer rate than others (i.e. the terms involving fluctuations of the temperature at the edge of the boundary layer).

For all cases studied, the trends were established over a wide range of fluctuations because no experimental data was available on the magnitudes of the fluctuations or the correlation factors for the cross terms. However, when a suitable method is found to accurately measure or predict the fluctuations, the results of this study can be applied directly to show the effects of not considering the fluctuations and calculating the heat transfer rate using the steady laminar Fay-Riddell equation. Additionally, the significant fluctuations can be included in the heat transfer rate calculations by the same modelling technique used in this study.

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APPENDIX A

ANALYTICAL DERIVATIVES OF THE TRANSPORT AND THERMODYNAMIC PROPERTIES

APPENDIX A

Analytical Derivatives of the Transport and Thermodynamic Properties

Enthalpy Derivatives

The expression used for the species enthalpy was

$$\frac{h_j - h_j^\circ}{T} = \frac{5+2(n_j-1)}{2} + \frac{(n_j-1)\,\Theta_{v_j}}{T(e^{\Theta_{v_j}/T}-1)} + \frac{\sum_{\ell=1}^{L_j} \varepsilon_{\ell_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}}{T \sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}} \qquad \text{(A-1)}$$

Normalizing the temperature with respect to T_{ref} and the energies with respect to $\bar{R}T_{ref}$ and solving for the species enthalpy

$$h_j = h_j^\circ + RT\left[\frac{5+2(n_j-1)}{2} + \frac{(n_j-1)\,\Theta_{v_j}}{T(e^{\Theta_{v_j}/T}-1)} + \frac{\sum_{\ell=1}^{L_j} \varepsilon_{\ell_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}}{T \sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}}\right] \qquad \text{(A-2)}$$

where the temperature and energies represent normalized values. The first derivative of the enthalpy with respect to temperature was expressed as

$$\frac{\partial h_j}{\partial T} = R\left[\frac{5+2(n_j-1)}{2} + (n_j-1)\left(\frac{\Theta_{v_j}}{T}\right)^2 \frac{e^{\Theta_{v_j}/T}}{(e^{\Theta_{v_j}/T}-1)^2} + \frac{\sum_{\ell=1}^{L_j} \left(\frac{\varepsilon_{\ell_j}}{T}\right)^2 g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}}{\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}} - \frac{\left[\sum_{\ell=1}^{L_j} \left(\frac{\varepsilon_{\ell_j}}{T}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right]^2}{\left[\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right]^2}\right] \qquad \text{(A-3)}$$

The second derivative of the enthalpy with respect to temperature was obtained by taking the derivative of each term in the brackets seperately. The derivative of the first term was zero. The derivative of the second was

expressed as

$$T_2 = (n_j-1)\left\{\left(\frac{\theta_{v_j}}{T}\right)^2 \left[\frac{\left(e^{\theta_{v_j}/T}-1\right)^2\left(e^{\theta_{v_j}/T}\right)\left(-\frac{\theta_{v_j}}{T^2}\right) - (2)\left(e^{\theta_{v_j}/T}-1\right)\left(e^{\theta_{v_j}/T}\right)\left(-\frac{\theta_{v_j}}{T^2}\right)\left(e^{\theta_{v_j}/T}\right)}{\left(e^{\theta_{v_j}/T}-1\right)^2}\right] - \left(\frac{2\theta_{v_j}^{\,2}}{T^3}\right)\left[\frac{e^{\theta_{v_j}/T}}{\left(e^{\theta_{v_j}/T}-1\right)^2}\right]\right\} \qquad \text{(A-4)}$$

After simplifying and rearranging, the second term was written as

$$T_2 = (n_j-1)\left(\frac{\theta_{v_j}^{\,2}}{T^3}\right)\left[\frac{e^{\theta_{v_j}/T}}{\left(e^{\theta_{v_j}/T}-1\right)^2}\right]\left[\left(\frac{\theta_{v_j}}{T}\right)\left(\frac{e^{\theta_{v_j}/T}+1}{e^{\theta_{v_j}/T}-1}\right) - 2\right] \qquad \text{(A-5)}$$

The derivative of the third term in Equation A-3 involving the electronic levels was

$$T_3 = \frac{\left[\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\epsilon_{\ell_j}/T}\right]\left[\sum_{\ell=1}^{L_j} g_{\ell_j}\left(\frac{\epsilon_{\ell_j}}{T}\right)^2 e^{-\epsilon_{\ell_j}/T}\left(\frac{\epsilon_{\ell_j}}{T^2}\right) + \sum_{\ell=1}^{L_j} g_{\ell_j}(-2)\left(\frac{\epsilon_{\ell_j}^{\,2}}{T^3}\right)\left(e^{-\epsilon_{\ell_j}/T}\right)\right]}{\left(\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\epsilon_{\ell_j}/T}\right)^2} - \frac{\left[\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\epsilon_{\ell_j}/T}\left(\frac{\epsilon_{\ell_j}}{T}\right)\right]\left[\sum_{\ell=1}^{L_j}\left(\frac{\epsilon_{\ell_j}}{T}\right)^2 g_{\ell_j} e^{-\epsilon_{\ell_j}/T}\right]}{\left(\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\epsilon_{\ell_j}/T}\right)^2} \qquad \text{(A-6)}$$

Simplifying the first part of this expression lead to

$$T_3 = \frac{\sum_{\ell=1}^{L_j} g_{\ell_j}\left(\frac{\epsilon_{\ell_j}}{T}\right)^2 e^{-\epsilon_{\ell_j}/T}\left(\frac{\epsilon_{\ell_j}}{T^2}\right) - 2\sum_{\ell=1}^{L_j} g_{\ell_j}\left(\frac{\epsilon_{\ell_j}^{\,2}}{T^3}\right)\left(e^{-\epsilon_{\ell_j}/T}\right)}{\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\epsilon_{\ell_j}/T}} - \frac{\left[\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\epsilon_{\ell_j}/T}\left(\frac{\epsilon_{\ell_j}}{T^2}\right)\right]\left[\sum_{\ell=1}^{L_j}\left(\frac{\epsilon_{\ell_j}}{T}\right)^2 g_{\ell_j} e^{-\epsilon_{\ell_j}/T}\right]}{\left(\sum_{\ell=1}^{L_j} g_{\ell_j} e^{-\epsilon_{\ell_j}/T}\right)^2} \qquad \text{(A-7)}$$

The derivative of the fourth term in Equation A-3, which also involves the electronic levels, was

$$T_4 = \frac{\left[\sum\limits_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right]^2 \left[2\left(\sum\limits_{\ell=1}^{L_j}\left(\frac{\varepsilon_{\ell_j}}{T}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right)\left(\sum\limits_{\ell=1}^{L_j}\left(\frac{\varepsilon_{\ell_j}}{T}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\left(\frac{\varepsilon_{\ell_j}}{T^2}\right) + \sum\limits_{\ell=1}^{L_j}(-1)\left(\frac{\varepsilon_{\ell_j}}{T^2}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right)\right]}{\left(\sum\limits_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right)^4}$$

$$- \frac{2\left(\sum\limits_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right)\left(\sum\limits_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\frac{\varepsilon_{\ell_j}}{T^2}\right)\left(\sum\limits_{\ell=1}^{L_j}\left(\frac{\varepsilon_{\ell_j}}{T}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right)^2}{\left(\sum\limits_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right)^4} \qquad \text{(A-8)}$$

Simplifying this expression lead to

$$T_4 = \frac{2\left[\sum\limits_{\ell=1}^{L_j}\left(\frac{\varepsilon_{\ell_j}}{T}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right]\left[\sum\limits_{\ell=1}^{L_j}\left(\frac{\varepsilon_{\ell_j}}{T}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T} - \sum\limits_{\ell=1}^{L_j}\left(\frac{\varepsilon_{\ell_j}}{T^2}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right]}{\left(\sum\limits_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right)^2}$$

$$- \frac{2\left[\sum\limits_{\ell=1}^{L_j}\left(\frac{\varepsilon_{\ell_j}}{T^2}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right]\left[\sum\limits_{\ell=1}^{L_j}\left(\frac{\varepsilon_{\ell_j}}{T}\right) g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right]^2}{\left(\sum\limits_{\ell=1}^{L_j} g_{\ell_j} e^{-\varepsilon_{\ell_j}/T}\right)^3} \qquad \text{(A-9)}$$

Now having the derivative of each term, the second derivative of the enthalpy was expressed as

$$\frac{\partial^2 h}{\partial T^2} = T_2 + T_3 + T_4 \qquad \text{(A-10)}$$

Prandtl Number Derivatives

The Prandtl Number was determined using curve fit data for three ranges of the ratio of the wall enthalpy and a reference enthalpy. For $h_w/h_o < 0.005$, the Prandtl Number was set equal to 0.77. Therefore, for this range, the first and second derivatives of Pr were equal to zero.

For $0.005 \le h_w/h_o \le 0.0$, the Prandtl Number was expressed by

$$P_r = P_1 - P_2\left(\frac{h_w}{h_o}\right) + P_3\left(\frac{h_w}{h_o}\right)^2 - P_4\left(\frac{h_w}{h_o}\right)^3 + P_5\left(\frac{h_w}{h_o}\right)^4 - P_6\left(\frac{h_w}{h_o}\right)^5 \qquad \text{(A-11)}$$

where P_1 through P_6 represented the coefficients of Equation 20. The first and second derivatives of the Prandtl Number were

$$\frac{\partial P_r}{\partial T_w} = \left[-\frac{P_2}{h_o} + \frac{2P_3}{h_o^2} h_w - \frac{3P_4}{h_o^3} h_w^2 + \frac{4P_5}{h_o^4} h_w^3 - \frac{5P_6}{h_o^5} h_w^4 \right] \left[\frac{\partial h_w}{\partial T_w} \right] \qquad \text{(A-12)}$$

$$\frac{\partial^2 P_r}{\partial T_w^2} = \left[\frac{2P_3}{h_o^2} - \frac{6P_4}{h_o^3} h_w + \frac{12P_5}{h_o^4} h_w^2 - \frac{20P_6}{h_o^5} h_w^3 \right] \left[\frac{\partial h_w}{\partial T_w} \right]^2$$

$$+ \left[-\frac{P_2}{h_o} + \frac{2P_3}{h_o^2} h_w - \frac{3P_4 h_w^2}{h_o^3} + \frac{4P_5}{h_o^4} h_w^3 - \frac{5P_6}{h_o^5} h_w^4 \right] \left[\frac{\partial^2 h_w}{\partial T_w^2} \right] \qquad \text{(A-13)}$$

For $0.1 \le h_w/h_o \le 2.0$, the Prandtl Number was expressed as

$$Pr = P_7 - P_8 \left(\frac{h_w}{h_o} \right) + P_9 \left(\frac{h_w}{h_o} \right)^2 + P_{10} \left(\frac{h_w}{h_o} \right)^3 \qquad \text{(A-14)}$$

where P_7 through P_{10} represent the coefficients of Equation 21. The first and second derivatives of the Prandtl Number for this range were

$$\frac{\partial P_r}{\partial T_w} = \left[-\frac{P_8}{h_o} + \frac{2P_9}{h_o^2} h_w + \frac{3P_{10}}{h_o^3} h_w^2 \right] \left[\frac{\partial h_w}{\partial T_w} \right] \qquad \text{(A-15)}$$

$$\frac{\partial^2 P_r}{\partial T_w^2} = \left[\frac{2P_9}{h_o^2} + \frac{6P_{10}}{h_o^3} h_w \right] \left[\frac{\partial h_w}{\partial T_w} \right]^2 + \left[-\frac{P_8}{h_o} + \frac{2P_9}{h_o^2} h_w + \frac{3P_{10}}{h_o^3} h_w^2 \right] \left[\frac{\partial^2 h_w}{\partial T_w^2} \right] \qquad \text{(A-16)}$$

The enthalpy derivatives were shown previously in this section.

Viscosity Derivatives

The viscosity was determined by the Sutherland Formula.

$$\mu = 2.27 \times 10^{-8} \frac{T^{1.5}}{T + 198.6} \qquad \text{(A-17)}$$

The first and second derivatives of the viscosity with respect to temperature were

$$\frac{\partial \mu}{\partial T} = 2.27 \times 10^{-8} \left[\frac{0.5\,T^{1.5} + (1.5)(198.6)\,T^{0.5}}{(T + 198.6)^2} \right] \qquad \text{(A-18)}$$

$$\frac{\partial^2 \mu}{\partial T^2} = 2.27 \times 10^{-8} \left\{ \left[\frac{0.75\,T^{0.5} + 0.75(198.6)\,T^{-0.5}}{(T + 198.6)^2} \right] - 2\left[\frac{0.5\,T^{1.5} + 1.5(198.6)\,T^{0.5}}{(T + 198.6)^3} \right] \right\} \qquad \text{(A-19)}$$

<u>Lewis Number Derivatives</u>

The Lewis Number was determined as a function of the ratio of the enthalpy at the edge of the boundary layer and a reference enthalpy. If $h_s/h_o < 0.1$, the Lewis Number was set equal to 1.4. Therefore, the first and second derivatives of the Lewis Number were set equal to zero. If $h_s/h_o \geq 0.1$, the Lewis Number was expressed by Equation 22 and the first and second derivatives were

$$\frac{\partial Le}{\partial T_s} = \left[-\frac{0.54919}{h_o} - \frac{(2)(0.07176)}{h_o^2} h_s + \frac{(3)(0.06833)}{h_o^3} h_s^2 \right] \left[\frac{\partial h_s}{\partial T_s} \right] \qquad \text{(A-20)}$$

$$\frac{\partial^2 Le}{\partial T_s^2} = \left[-\frac{0.54919}{h_o} - \frac{(2)(0.07176)}{h_o^2} h_s + \frac{(3)(0.06833)}{h_o^3} h_s^2 \right] \left[\frac{\partial^2 h_s}{\partial T_s^2} \right]$$

$$+ \left[-\frac{(2)(0.07176)}{h_o^2} + \frac{(6)(0.06833)}{h_o^3} h_s \right] \left[\frac{\partial h_s}{\partial T_s} \right]^2 \qquad \text{(A-21)}$$

The enthalpy derivatives were shown previously in this section.

h_D/h_s Derivatives

The h_D/h_s ratio was determined using curve fit data as a function of the enthalpy at the edge of the boundary in BTU/lb. The ratio was expressed as

$$\frac{h_D}{h_s} = -H_1 + H_2 h_s - H_3 h_s^2 + H_4 h_s^3 - H_5 h_s^4 \qquad \text{(A-22)}$$

where H_1 through H_5 represented the coefficients of Equation 23. The first and second derivatives of the ratio were

$$\frac{\partial\left(\frac{h_D}{h_s}\right)}{\partial T_s} = \left(H_2 - 2H_3 h_s + 3H_4 h_s^2 - 4H_5 h_s^3\right)\left(\frac{\partial h_s}{\partial T_s}\right) \qquad \text{(A-23)}$$

$$\frac{\partial^2\left(\frac{h_D}{h_s}\right)}{\partial T_s^2} = \left(H_2 - 2H_3 h_s + 3H_4 h_s^2 - 4H_5 h_s^3\right)\left(\frac{\partial^2 h_s}{\partial T_s^2}\right)$$

$$+\left(-2H_3 + 6H_4 h_s - 12H_5 h_s^2\right)\left(\frac{\partial h_s}{\partial T_s}\right)^2 \qquad \text{(A-24)}$$

APPENDIX B

COMPUTER PROGRAM

APPENDIX B

Computer Program

Explanation of Program

The Fortran computer program incorporates the equations derived in Chapters II and III to determine the effect of fluctuations of the five independent variables on the heat transfer rate as calculated by the Fay-Riddell equation. In addition to the main program, there are numerous subroutines.

The main program contains the calling sequence for the various subroutines. The heat transfer rate without fluctuations is calculated in this part of the program. Also, the second derivatives that do not involve cross terms are computed after the necessary parameters are determined from other subroutines. These values are printed out in the main program.

Subroutine READ initializes the five independent variables and the desired fluctuations. The fluctuations are normalized by the initial values of the independent variables. These values are also printed out in this subroutine.

Subroutine THERM computes the enthalpy of the mixture after computing the enthalpy of each species. Although only the nitrogen and oxygen molecules are considered in this study, the subroutine is written to allow for easy inclusion of other species. The temperatures and energies

are normalized for the enthalpy computations. The enthalpy of the mixture is returned to the main program in the units of ft^2/sec^2. Subroutine HDE computes the first and second derivatives of the enthalpy in the same manner. The temperatures and energies are also normalized to calculate these derivatives.

Subroutine VISCTY determines the viscosity at the wall and the boundary layer as a function of the respective temperatures. The viscosity is returned to the main program in the units of slugs/ft-sec. Subroutine XMUDE computes the first and second derivatives of the viscosities.

Subroutine PRANUM and LEWIS calculate the Prandtl Number and Lewis Number from the curve fit data shown in Chapter III. The reference enthalpy used in both subroutines is 2.119×10^8 ft^2/sec^2 (Ref 11). Subroutines PRADE and XLEWDE compute the first and second derivatives of the Prandtl Number and Lewis Number, respectively.

Subroutine RATIO computes the ratio of the dissociation energy and the enthalpy at the edge of the boundary layer. The enthalpy is converted to BTU/lb for the curve fit data used in this subroutine. The first and second derivatives of this ratio are computed in subroutine HRATDE.

Subroutines ALPHDE, ADE, BETDE, and BEEDE calculate the first and second derivatives of α, A, β, and B, respectively. These values are calculated from the initial conditions and the other derivatives previously computed. The magnitudes of the derivatives are returned to the main

program and used to compute the derivatives of the heat transfer rate with respect to the temperature at the wall and the temperature at the edge of the boundary layer.

Subroutines GAWADE, GASDE, and GAPTDE compute the first and second derivatives of γ with respect to the density at the wall, density at the edge of the boundary layer, and the stagnation pressure. The values of these derivatives are returned to the main program and used to compute the second derivatives of the heat transfer rate with respect to the same three independent variables.

Subroutines CROSS computes all of the second derivatives of the heat transfer rate that involve cross terms of the independent variables. The value of these derivatives are also written out in this subroutine.

Subroutine NORM normalizes and non-dimensionalizes all of the second derivatives of the heat transfer rate. They are normalized by dividing each term by the magnitude of the heat transfer rate without fluctuations. The values of the normalized derivatives are then written out. The derivatives are non-dimensionalized by multiplying each second derivative by the magnitude of the independent variable or variables that the derivative was taken with respect to. The values of the normalized, non-dimensionalized derivatives are also printed out in this subroutine.

In subroutine FLUCTS, the normalized, non-dimensionalized second derivatives of the heat transfer rate are multiplied by the magnitude of the desired fluctuations. These

terms are summed to obtain the heat transfer rate from the Taylor Series expansion. The exact and normalized value of the heat transfer rate with fluctuations are printed out in this subroutine. Additionally, a check is conducted to ensure the full Taylor Series expansion accurately approximates the exact value of the heat transfer rate for the fluctuations considered.

Fortran Variables

The major Fortran variables and the definition of each are listed in Table II. Additionally, the second derivatives of the heat transfer rate are expressed by the letters D2Q followed by the independent variables that the derivative was taken with respect to (i.e. the second derivative of the heat transfer rate with respect to the wall temperature and density is D2QTWRW).

The first and second derivatives of the other flow properties have, in addition to the letters used for the flow property itself, a D for the derivative and a 1 or 2 denoting which derivative is represented.

The fluctuations are in matrix form of dimension EPS (5,5). The diagonal elements represent the fluctuations of only one variable and the off-diagonal elements represent the cross-term fluctuations.

A listing of the Fortran program follows in this Appendix.

Table II

Definitions of the Major Fortran Variables

Variable (Dimensions)	Definition (Units)
ATOMS (2)	Number of atoms for each species
DENS	Density at edge of boundary layer (slugs/ft^3)
DENWAL	Density at wall (slugs/ft^3)
ENTHS	Enthalpy at the edge of the boundary layer (ft^2/sec^2)
ENTHW	Enthalpy at the wall (ft^2/sec^2)
ESPL (2,6)	Energy of each electronic level for each species (cal/mole)
GL (2,6)	Degeneracy of each electronic level for each species
HDTS 1,2	First and second derivatives of enthalpy at the edge of the boundary layer
HDTW 1,2	First and second derivatives of wall enthalpy
HFORM (2)	Enthalpy of formation (cal/mole)
HRATIO	Ratio of dissociation energy and enthalpy
PR	Prandtl Number
PR1-PR10	Coefficients for Prandtl Number curve fits
PRE	Stagnation pressure (lb/ft^2)

Table II - Continued

Variable (Dimensions)	Definition (Units)
QAVG	$q\sqrt{R}$ without fluctuations (BTU/$ft^{1.5}$-sec)
QDOT	$q\sqrt{R}$ with fluctuations (BTU/$ft^{1.5}$-sec)
R	Universal gas constant
THETA (2)	Characteristic vibrational temperature (^{o}R)
TS	Temperature at edge of boundary layer (^{o}R)
TW	Temperature at wall (^{o}R)
XLEWA-XLEWD	Coefficients for Lewis Number curve fit
XLEWIS	Lewis Number
XMUS	Viscosity at the edge of the boundary layer (slugs/ft-sec)
XMUW	Viscosity at the wall (slugs/ft-sec)

```
      PROGRAM HEAT (INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT)
C
C     THIS PROGRAM DETERMINES THE EFFECT OF FLOW PROPERTY FLUCTUATIONS ON THE
C     STAGNATION POINT HEAT TRANSFER RATE AS CALCULATED BY THE FAY-RIDDELL EQ.
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
      COMMON/F/FFS(5,5),KK
      COMMON/KROS/D2QTWTS,D2QTWRW,D2QTWRS,D2QTWPT,D2QRWTS,D2QTSRS,D2QTSP
     1T,D2QRWRS,D2QRWPT,D2QRSPT
      CALL READ (TW,TS,DENS,DENWAL,PRE)
      R=1.987
C     NON-DIMENSIONALIZES TEMPERATURES AND ENERGIES FOR ENTHALPY CALCULATIONS
      TSK=TS/1.8
      DO 40 I=1,2
   40 THETA(I)=THETA(I)/TSK
      DO 180 I=1,2
      DO 180 J=1,6
  180 ESPL(I,J)=ESPL(I,J)/(R*TSK)
      CALL THERM (TW,TSK,ENTHW)
      CALL THERM (TS,TSK,ENTHS)
      CALL PRANUM (ENTHW,PR)
      CALL VISCTY(TW,XMUW)
      CALL VISCTY(TS,XMUS)
      CALL LEWIS (ENTHS,XLEWIS)
      CALL RATIO (ENTHS,HRATIO)
C     CALCULATION OF HEAT TRANSFER RATE WITHOUT FLUCTUATIONS
      QAVG=0.763*(PR**(-0.6))*((DENWAL*XMUW)**0.1)*((DENS*XMUS)**0.4)*(1
     11.0+(((XLEWIS**0.52)-1.0)*HRATIO))*(ENTHS-ENTHW)*(((2.0*PRE)/DENS)*
     2*0.25)*(1.0/777.66)
      WRITE (6,120) ENTHS,ENTHW,XMUW,XMUS,PR,XLEWIS,HRATIO,QAVG
  120 FORMAT (10X,"B.L.ENTHALPY=",E12.4/,10X,"WALL ENTHALPY=",E12.4/,10X
     1,"WALL VISCOSITY=",E12.4/,10X,"B.L. VISCOSITY=",E12.4/,10X,"PR=",E
     212.4/,10X,"LEWIS=",E12.4/,10X,"HRATIO=",E12.4/,10X,"QDOT*SQRT(R)="
```

```
     3,E12.4)
      ALPH=(XMUS**0.4)*(1.0+(((XLEWIS**0.52)-1.0)*(HRATIO)))
      A=ENTHS*ALPH
      BETA=(PR**(-0.6))*(XMUW**0.1)
      B=ENTHW*BETA
      GAM=0.907365*(DENWAL**0.1)*(DENS**0.15)*(PRE**0.25)
      CALL HDE  (TW,TSK,HDTW1,HDTW2)
      CALL XMUDE  (TW,XMUDW1,XMUDW2)
      CALL PRADE  (ENTHW,HDTW1,HDTW2,PRD1,PRD2)
      CALL BETDE(PR,XMUW,XMUDW1,XMUDW2,PRD1,PRD2,BETD1,BETD2)
      CALL BEEDE(ENTHW,BETA,BETD1,BETD2,HDTW1,HDTW2,BEED1,BEED2)
      QTWD1=GAM*((A*BETD1)-(ALPH*BEED1))/777.66
      QTWD2=GAM*((A*BETD2)-(ALPH*BEED2))/777.66
      CALL HDE(TS,TSK,HDTS1,HDTS2)
      CALL HRATDE(ENTHS,HDTS1,HDTS2,HRATD1,HRATD2)
      CALL XMUDE(TS,XMUDS1,XMUDS2)
      CALL XLEWDE  (ENTHS,HDTS1,HDTS2,XLEWD1,XLEWD2)
      CALL ALPHDE(XMUS,XMUDS1,XMUDS2,XLEWIS,XLEWD1,XLEWD2,HRATD1,HRATD2,
     1HRATIO,ALPHD1,ALPHD2)
      CALL ADE(ENTHS,HDTS1,HDTS2,ALPH,ALPHD1,ALPHD2,AD1,AD2)
      QTSD1=GAM*((BETA*AD1)-(ALPHD1*B))/777.66
      QTSD2=GAM*((BETA*AD2)-(ALPHD2*B))/777.66
      CALL GAWADE(DENWAL,DENS,PRE,GAWAD1,GAWAD2)
      QROWD1=GAWAD1*((BETA*A)-(ALPH*B))/777.66
      QROWD2=GAWAD2*((BETA*A)-(ALPH*B))/777.66
      CALL GASDE(DENWAL,DENS,PRE,GASD1,GASD2)
      QROSD1=GASD1*((BETA*A)-(ALPH*B))/777.66
      QROSD2=GASD2*((BETA*A)-(ALPH*B))/777.66
      CALL GAPTDE(DENWAL,DENS,PRE,GAPTD1,GAPTD2)
      QPTD1=GAPTD1*((BETA*A)-(ALPH*B))/777.66
      QPTD2=GAPTD2*((BETA*A)-(ALPH*B))/777.66
      WRITE(6,930) QTWD2,QTSD2,QROWD2,QROSD2,QPTD2
  930 FORMAT(10X,"QTWD2=",E15.8/,10X,"QTSD2=",E15.8/,10X,"QROWD2=",E15.8
     1/,10X,"QROSD2=",E15.8/,10X,"QPTD2=",E15.8/)
      CALL CROSS  (A,B,ALPH,BETA,GAM,AD1,BETD1,ALPHD1,BEED1,GAWAD1,GASD1,
```

```
     1GAPTD1,DENS,DENWAL,PRE)
      CALL NORM (QAVG,QTWD2,QTSD2,QROWD2,QROSD2,QPTD2,TW,TS,DENS,DENWAL,
     1PRE)
      CALL FLUCTS(QAVG,QTWD2,QTSD2,QROWD2,QROSD2,QPTD2,QTWD1,QTSD1,QROWD
     11,QROSD1,QPTD1,TW,TS,DENWAL,DENS,PRE,TSK,P)
      STOP
      END

************************************************************************
************************************************************************

      SUBROUTINE READ(TW,TS,DENS,DENWAL,PRE)
C
C     INITIALIZES THE FIVE INDEPENDENT VARIBLES AND THE FLUCTUATIONS
C     (TEMPERATURES IN DEGREES RANKINE,DENSITIES IN SLUGS/FT3,STAGNATION
C     PRESSURE IN LB/FT2)
C
      COMMON/F/EPS(5,5),KK
      READ (5,110) TW,TS,DENWAL,DENS,PRE
  110 FORMAT (F12.4,F12.4,F12.6,F12.6,F12.4)
      WRITE(6,900) TW,TS,DENWAL,DENS,PRE
  900 FORMAT(10X,"TW=",E15.4/,10X,"TS=",E15.4/,10X,"RHOW=",E15.4/,10X,"R
     1HOS=",E15.4/,10X,"PT2=",E15.4///)
      READ(5,111) (EPS(I,I),I=1,5)
  111 FORMAT(5F10.3)
      READ(5,112) (EPS(1,I),I=2,5)
  112 FORMAT(4F10.3)
      READ(5,113) (EPS(2,I),I=3,5)
  113 FORMAT(3F10.3)
      READ(5,114) EPS(3,4),EPS(3,5),EPS(4,5)
  114 FORMAT(3F10.3)
      WRITE(6,901) (EPS(I,I),I=1,5)
  901 FORMAT(10X,"ERHOS=",F10.3/,10X,"ERHOW=",F10.3/,10X,"ETS=",F12.3/,1
     10X,"ETW=",F12.3/,10X,"EPT2=",F11.3)
      WRITE(6,902) (EPS(1,I),I=2,5)
```

```
902 FORMAT(10X,"ERWRS=",F10.3/,10X,"ETSRS=",F10.3/,10X,"ETWRS=",F10.3/
   1,10X,"ERSPT=",F10.3)
    WRITE(6,903) (EPS(2,I),I=3,5)
903 FORMAT(10X,"ERWTS=",F10.3/,10X,"ETWRW=",F10.3/,10X,"ERWPT=",F10.3)
    WRITE(6,904) EPS(3,4),EPS(3,5),EPS(4,5)
904 FORMAT(10X,"ETWTS=",F10.3/,10X,"ETSPT=",F10.3/,10X,"ETWPT=",F10.3/
   1///)
    RETURN
    END

*****************************************************************************

      SUBROUTINE THERM (T,TSK,ENTHAL)
C
C     CALCULATION OF THE ENTHALPY IN FT2/SEC2
C
      DIMENSION ELECL(2),PTENT(2),XNUM(2),DENOM(2),PNUM(2,6),PDENOM(2,6)
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
C     CONV IS THE CONVERSION FROM CAL/GM-MOLE TO FT2/SEC2
      CONV=45073.968
      R=1.987
C     TEMPERATURE CONVERTED TO DEGREES KELVIN
      T=T/1.8
      T=T/TSK
      ENTHAL=0.0
      DO 250 I=1,2
      ELECL(I)=0.0
      XNUM(I)=0.0
  250 DENOM(I)=0.0
      DO 200 I=1,2
      DO 210 L=1,6
      PNUM(I,L)=ESPL(I,L)*GL(I,L)*EXP(-ESPL(I,L)/T)
      XNUM(I)=XNUM(I)+PNUM(I,L)
```

```
      PDENOM(I,L)=GL(I,L)*EXP(-ESPL(I,L)/T)
  210 DENOM(I)=DENOM(I)+PDENOM(I,L)
  200 ELECL(I)=XNUM(I)/(T*DENOM(I))
      DO 220 I=1,2
      PTENT(I)=HFORM(I)+R*T*TSK*(((5.0+2.0*(ATOMS(I)-1.0))/2.0)+(((ATOMS
     1(I)-1.0)*THETA(I))/(T*(EXP(THETA(I)/T)-1.0)))+ELECL(I))*CONV
  220 ENTHAL=ENTHAL+X(I)*PTENT(I)
      ENTHAL=ENTHAL/28.8
C     TEMPERATURE CONVERTED BACK TO DEGREES RANKINE
      T=T*1.8*TSK
      RETURN
      END

****************************************************************************

      SUBROUTINE PRANUM (ENTHW,PR)
C
C     CALCULATION OF PRANDTL NUMBER
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
      HO=2.119E+08
      HWR=ENTHW/HO
      IF (HWR.GE.0.005) GO TO 300
      PR=0.770
      GO TO 390
  300 IF (HWR.GT.0.1) GO TO 310
      PR=PR1-PR2*HWR+PR3*(HWR**2.0)-PR4*(HWR**3.0)+PR5*(HWR**4.0)-PR6*(H
     1WR**5.0)
      GO TO 390
  310 PR=PR7-PR8*HWR+PR9*(HWR**2.0)+PR10*(HWR**3.0)
  390 CONTINUE
      RETURN
      END
```

```
************************************************************************
      SUBROUTINE VISCTY (T,XMU)
C
C     CALCUTION OF VISCOSITY (SLUGS/FT-SEC)
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
      XMU=VISCA*((T**1.5)/(T+VISCB))
      RETURN
      END
************************************************************************
      SUBROUTINE LEWIS (ENTHS,XLEWIS)
C
C     CALCULATION OF LEWIS NUMBER
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
      XLEWIS=XLEWA-XLEWB*ENTHS-XLEWC*(ENTHS**2.0)+XLEWD*(ENTHS**3.0)
      RETURN
      END
************************************************************************
      SUBROUTINE RATIO (ENTHS,HRATIO)
C
C     COMPUTES HD/HS RATIO
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
```

```
      2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
C      CONVERTS ENTHS TO BTU/LB
       ENTHS=ENTHS/(32.2*777.66)
       HRATIO=-H1+H2*ENTHS-H3*(ENTHS**2.0)+H4*(ENTHS**3.0)-H5*(ENTHS**4.0
      1)
C      CONVERTS ENTHS BACK TO FT2/SEC2
       ENTHS=ENTHS*32.2*777.66
       RETURN
       END

*******************************************************************************

       BLOCK DATA INPT
C
C      INITIALIZES CONSTANTS REQUIRED FOR CALCULATION OF FLOW PROPERTIES
C
       COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
      1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
      2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
       DATA HFORM,ATOMS/0.0,0.0,2.0,2.0/
       DATA THETA/3353.24,2238.97/
       DATA ESPL/0.0,0.0,1.43685E+05,2.2637E+04,1.70475E+05,3.7725E+04,1.
      17154E+05,1.03198E+05,0.0,1.4239E+05,0.0,0.0/
       DATA GL/1.0,3.0,3.0,2.0,6.0,1.0,1.0,3.0,0.0,3.0,0.0,0.0/
       DATA VISCA,VISCB/2.27E-03,1.986E+02/
       DATA XLEWA,XLEWB,XLEWC,XLEWD/1.43691,2.59174E-09,1.59816E-18,7.181
      155E-27/
       DATA HD/3.14254E+08,1.88597E+08/
       DATA X/0.80,0.20/
       DATA XMW/28.014,32.000/
       DATA PR1,PR2,PR3,PR4,PR5,PR6/0.8245251,0.1212104E+02,0.349395E+03,
      10.4212513E+04,0.2404883E+05,0.5307031E+05/
       DATA PR7,PR8,PR9,PR10/0.788199,0.128764,0.238641E-01,0.102987E-01/
       DATA H1,H2,H3,H4,H5/0.12455,0.227850E-03,0.294417E-07,0.18460E-11,
      10.432111E-16/
```

```
      END
*******************************************************************************
      SUBROUTINE HDE(T,TSK,XHDT1,XHDT2)
C
C     CALCULATION OF FIRST AND SECOND DERIVATIVES OF ENTHALPY
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
      DIMENSION TERM11(2),TERM12(2),TERM13(2),TERM14(2),TERM21(2),TERM22
     1(2),TERM23(2),PTEND1(2),PTEND2(2)
C     CALCULATION OF FIRST DERIVATIVE
      DATA XNU13,XNU14,DEN13,DEN14/0.0,0.0,0.0,0.0/
      XHDT1=0.0
      XHDT2=0.0
      T=T/1.8
      T=T/TSK
      R=1.987
      CONV1=25041.0936
      CONV2=13911.71869
      DO 230 IA=1,2
      TERM11(IA)=(5.0+2.0*(ATOMS(IA)-1.0))/2.0
      W=THETA(IA)/T
  230 TERM12(IA)=(ATOMS(IA)-1.0)*(W**2.0)*(EXP(W)/((EXP(W)-1.0)**2.0))
      DO 231 IB=1,2
      DO 232 LB=1,6
      F=ESPL(IB,LB)/T
      Z=GL(IB,LB)*EXP(-F)
      PNU13=Z*(F**2.0)
      XNU13=XNU13+PNU13
  232 DEN13=DEN13+Z
  231 TERM13(IB)=XNU13/DEN13
      DO 233 IC=1,2
```

```
      DO 234 LC=1,6
      F=ESPL(IC,LC)/T
      Z=GL(IC,LC)*EXP(-F)
      FNU14=Z*F
      XNU14=XNU14+FNU14
  234 DEN14=DEN14+Z
  233 TERM14(IC)=(XNU14**2.0)/(DEN14**2.0)
      DO 235 ID=1,2
      PTEND1(ID)=R*(TERM11(ID)+TERM12(ID)+TERM13(ID)+TERM14(ID))*(CONV1)
  235 XHDT1=XHDT1+X(ID)*PTEND1(ID)
      XHDT1=XHDT1/28.8
C     CALCULATION OF SECOND DERIVATIVE
      DATA XNU22A,XNU22B,XNU22C,XNU22D,DEN22,XNU23A,XNU23B,XNU23C,DEN23/
     10.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0/
      DO 261 JB=1,2
      W=THETA(JB)/T
  261 TERM21(JB)=(ATOMS(JB)-1.0)*((W**2.0)/T)*(EXP(W)/((EXP(W)-1.0)**2.0
     1))*(W*((EXP(W)+1.0)/(EXP(W)-1.0))-2.0)
      DO 262 ID=1,2
      DO 263 LD=1,6
      F=ESPL(ID,LD)/T
      Z=GL(ID,LD)*EXP(-F)
      FNU22A=Z*((F**3.0)/T)
      XNU22A=XNU22A+FNU22A
      FNU22B=Z*((F**2.0)/T)
      XNU22B=XNU22B+FNU22B
      FNU22C=Z*(F/T)
      XNU22C=XNU22C+FNU22C
      FNU22D=Z*(F**2.0)
      XNU22D=XNU22D+FNU22D
  263 DEN22=DEN22+Z
  262 TERM22(ID)=((XNU22A-2.0*XNU22B)/DEN22)-((XNU22C*XNU22D)/(DEN22**2.
     10))
      DO 264 IE=1,2
      DO 265 LE=1,6
```

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```
      F=ESPL(IE,LE)/T
      Z=GL(IE,LE)*EXP(-F)
      FNU23A=Z*F
      XNU23A=XNU23A+FNU23A
      FNU23B=Z*((F**2.0)/T)
      XNU23B=XNU23B+FNU23B
      FNU23C=Z*(F/T)
      XNU23C=XNU23C+FNU23C
  265 DEN23=DEN23+Z
  264 TERM23(IE)=(((2.0*XNU23A)*(XNU23B-XNU23C))/(DEN23**2.0))-(((2.0*XN
     1U23C)*(XNU23A**2.0))/(DEN23**3.0))
      DO 266 IG=1,2
      PTEND2(IG)=(R/TSK)*(TERM21(IG)+TERM22(IG)+TERM23(IG))*(CONV2)
  266 XHDT2=XHDT2+X(IG)*PTEND2(IG)
      XHDT2=XHDT2/28.8
      T=T*1.8*TSK
      RETURN
      END

*********************************************************************************

      SUBROUTINE XMUDE(T,XMUD1,XMUD2)
C
C     CALCULATION OF FIRST AND SECOND DERIVATIVES OF VISCOSITY
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
C     CALCULATION OF FIRST DERIVATIVE
      XNUM1=(0.5*(T**1.5))+(1.5*VISCB*(T**0.5))
      XMUD1=(VISCA*XNUM1)/((T+VISCB)**2.0)
C     CALCULATION OF SECOND DERIVATIVE
      XNUM1A=((T+VISCB)**2.0)*((0.75*(T**0.5))+(0.75*VISCB*(T**(-0.5))))
      XNUM1B=((0.5*(T**1.5))+(1.5*VISCB*(T**0.5)))*(2.0*(T+VISCB))
      XMUD2=VISCA*((XNUM1A-XNUM1B)/((T+VISCB)**4.0))
```

```
      RETURN
      END
C**********************************************************************
      SUBROUTINE PRADE(ENTHW,HDTW1,HDTW2,PRD1,PRD2)
C
C     CALCULATION OF FIRST AND SECOND DERIVATIVES OF PRANDTL NUMBER
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
      HO=2.119E+08
      HWR=ENTHW/HO
      IF(HWR.GE.0.005) GO TO 510
      PRD1=0.0
      PRD2=0.0
      GO TO 519
  510 IF (HWR.GT.0.1) GO TO 512
      PRD1=((-PR2/HO)+((2.0*PR3*HWR)/HO)-((3.0*PR4*(HWR**2.0))/HO)+((4.0
     1*PR5*(HWR**3.0))/HO)-((5.0*PR6*(HWR**4.0))/HO))*(HDTW1)
      TERM2A=(((2.0*PR3)/HO**2.0)-((6.0*PR4*HWR)/(HO**2.0))+((12.0*PR5*(
     1HWR**2.0))/(HO**2.0))-((20.0*PR6*(HWR**3.0))/(HO**2.0)))*(HDTW1**2
     2.0)
      TERM2B=((-PR2/HO)+((2.0*PR3*HWR)/HO)-((3.0*PR4*(HWR**2.0))/HO)+((4
     1.0*PR5*(HWR**3.0))/HO)-((5.0*PR6*(HWR**4.0))/HO))*(HDTW2)
      PRD2=TERM2A+TERM2B
      GO TO 519
  512 PRD1=((-PR8/HO)+((2.0*PR9*HWR)/HO)*((3.0*PR10*(HWR**2.0))/HO))*(HD
     1TW1)
      TERM2A=(((2.0*PR9)/(HO**2.0))+((6.0*PR10*HWR)/(HO**2.0)))*(HDTW1**
     12.0)
      TERM2B=((-PR8/HO)+((2.0*PR9*HWR)/HO)+((3.0*PR10*(HWR**2.0))/HO))*(
     1HDTW2)
      PRD2=TERM2A+TERM2B
```

```
519 CONTINUE
    RETURN
    END

******************************************************************************

    SUBROUTINE BETDE(PR,XMUW,XMUDW1,XMUDW2,PRD1,PRD2,BETD1,BETD2)
C
C   CALCULATION OF BETA DERIVATIVES
C
C   CALCULATION OF FIRST DERIVATIVE
    BETD1=(0.1*(PR**(-0.6))*(XMUW**(-0.9))*XMUDW1)-(0.6*(PR**(-1.6))*(
   1XMUW**0.1)*PRD1)
C   CALCULATION OF SECOND DERIVATIVE
    TERM2A=(-0.06*(PR**(-1.6))*PRD1*(XMUW**(-0.9))*XMUDW1)-(0.09*(PR**
   1(-0.6))*(XMUW**(-1.9))*(XMUDW1**2.0))+(0.1*(PR**(-0.6))*(XMUW**(-0
   2.9))*XMUDW2)
    TERM2B=(0.96*(PR**(-2.6))*(PRD1**(2.0))*(XMUW**0.1))-(0.06*(PR**(-
   11.6))*(XMUW**(-0.9))*XMUDW1*PRD1)-(0.6*(PR**(-1.6))*(XMUW**0.1)*PR
   2D2)
    BETD2=TERM2A+TERM2B
    RETURN
    END

******************************************************************************

    SUBROUTINE BEEDE(ENTHW,BETA,BETD1,BETD2,HDTW1,HDTW2,BEED1,BEED2)
C
C   CALCULATION OF B DERIVATIVES
C
    BEED1=(ENTHW*BETD1)+(BETA*HDTW1)
    BEED2=(2.0*HDTW1*BETD1)+(ENTHW*BETD2)+(BETA*HDTW2)
    RETURN
    END
```

```
***********************************************************************

      SUBROUTINE HRATDE(ENTHS,HDTS1,HDTS2,HRATD1,HRATD2)
C
C     CALCULATION OF HD/HS RATIO DERIVATIVES
C
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
     2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
C     CONVERSION TO BTU/LB
      ENTHS=ENTHS/(32.2*777.66)
      HDTS1=HDTS1/(777.66*32.2)
      HDTS2=HDTS2/(777.66*32.2)
      HRATD1=(H2-(2.0*H3*ENTHS)+(3.0*H4*(ENTHS**2.0))-(4.0*H5*(ENTHS**3.
     10)))*(HDTS1)
      TERM2A=(H2-(2.0*H3*ENTHS)+(3.0*H4*(ENTHS**2.0))-(4.0*H5*(ENTHS**3.
     10)))*(HDTS2)
      TERM2B=((-2.0*H3)+(6.0*H4*ENTHS)-(12.0*H5*(ENTHS**2.0)))*(HDTS1**2
     1.0)
      HRATD2=TERM2A+TERM2B
C     CONVERSION TO FT2/SEC2
      ENTHS=ENTHS*32.2*777.66
      HDTS1=HDTS1*777.66*32.2
      HDTS2=HDTS2*777.66*32.2
      RETURN
      END

***********************************************************************

      SUBROUTINE XLEWDE(ENTHS,HDTS1,HDTS2,XLEWD1,XLEWD2)
C
C     CALCULATION OF LEWIS NUMBER DERIVATIVES
C
      HO=2.119E+08
      HSR=ENTHS/HO
```

```
      XLEWD1=((-0.54919/HO)-((2.0*0.07176*HSR)/HO)+((3.0*0.06833*(HSR**2
     1.0))/HO))*HDTS1
      TERM2A=((-0.54919/HO)-((2.0*0.07167*HSR)/HO)*((3.0*0.06833*(HSR**2
     1.0))/HO))*(HDTS2)
      TERM2B=(((-2.0*0.07176)/(HO**2.0))+((6.0*0.06833*HSR)/(HO**2.0)))*
     1(HDTS1**2.0)
      XLEWD2=TERM2A+TERM2B
      RETURN
      END
************************************************************************
      SUBROUTINE ALPHDE(XMUS,XMUDS1,XMUDS2,XLEWIS,XLEWD1,XLEWD2,HRATD1,H
     1RATD2,HRATIO,ALPHD1,ALPHD2)
C
C     CALCULATION OF ALPHA DERIVATIVES
C
      TERM1A=(0.4*(1.0/XMUS**0.6)*XMUDS1)*(1.0+(((XLEWIS**0.52)-1.0)*HRA
     1TIO))
      TERM1B=(XMUS**(0.4))*((XLEWIS**0.52)-1.0)*HRATD1
      TERM1C=0.52*(XMUS**0.4)*HRATIO*(XLEWIS**(-0.48))*XLEWD1
      ALPHD1=TERM1A+TERM1B+TERM1C
      TERM2A=-0.24*(1.0/XMUS**1.6)*(1.0+(((XLEWIS**0.52)-1.0)*HRATIO))*(
     1XMUDS1**2.0)
      TERM2B=0.416*(1.0/XMUS**0.6)*(XLEWIS**(-0.48))*HRATIO*XLEWD1*XMUDS
     11
      TERM2C=0.8*(1.0/XMUS**0.6)*((XLEWIS**0.52)-1.0)*HRATD1*XMUDS1
      TERM2D=0.4*(1.0/XMUS**0.6)*(1.0+(((XLEWIS**0.52)-1.0)*HRATIO))*XMU
     1DS2
      TERM2E=1.04*(XMUS**0.4)*(XLEWIS**(-0.48))*XLEWD1*HRATD1
      TERM2F=(XMUS**0.4)*((XLEWIS**0.52)-1.0)*HRATD2
      TERM2G=-0.2496*(XMUS**0.4)*HRATIO*(XLEWIS**(-1.48))*(XLEWD1**2.0)
      TERM2H=0.52*(XMUS**0.4)*HRATIO*(XLEWIS**(-0.48))*(XLEWD2)
      ALPHD2=TERM2A+TERM2B+TERM2C+TERM2D+TERM2E+TERM2F+TERM2G+TERM2H
      RETURN
```

```
      END

*******************************************************************************

      SUBROUTINE ADE(ENTHS,HDTS1,HDTS2,ALPH,ALPHD1,ALPHD2,AD1,AD2)
C
C     CALCULATION OF A DERIVATIVES
C
      AD1=(ENTHS*ALPHD1)+(HDTS1*ALPH)
      AD2=(ENTHS*ALPHD2)+(2.0*HDTS1*ALPHD1)+(ALPH*HDTS2)
      RETURN
      END

*******************************************************************************

      SUBROUTINE GAWADE(DENWAL,DENS,PRE,GAWAD1,GAWAD2)
C
C     CALCULATION OF GAMMA DERIVATIVES WITH RESPECT TO WALL DENSITY
C
      GAWAD1=0.0907365*(DENS**0.15)*(PRE**0.25)*(DENWAL**(-0.9))
      GAWAD2=-0.0816628*(DENS**0.15)*(PRE**0.25)*(DENWAL**(-1.9))
      RETURN
      END

*******************************************************************************

      SUBROUTINE GASDE(DENWAL,DENS,PRE,GASD1,GASD2)
C     CALCULATION OF GAMMA DERIVATIVES WITH RESPECT TO DENSITY AT EDGE OF
C     BOUNDARY LAYER
C
      GASD1=0.1361047*(DENWAL**0.1)*(PRE**0.25)*(DENS**(-0.85))
      GASD2=-0.115689*(DENWAL**0.1)*(PRE**0.25)*(DENS**(-1.85))
      RETURN
      END
```

```
************************************************************************

      SUBROUTINE GAPTDE(DENWAL,DENS,PRE,GAPTD1,GAPTD2)
C
C     CALCULATION OF GAMMA DERIVATIVES WITH RESPECT TO STAGNATION PRESSURE
C
      GAPTD1=0.226841*(DENWAL**0.1)*(DENS**0.15)*(PRE**(-0.75))
      GAPTD2=-0.1701309*(DENWAL**0.1)*(DENS**0.15)*(PRE**(-1.75))
      RETURN
      END

************************************************************************

      SUBROUTINE CROSS(A,B,ALPHA,BETA,GAM,AD1,BETD1,ALPHD1,BEED1,GAWAD1,
     1GASD1,GAPTD1,DENS,DENWAL,PRE)
C
C     CALCULATION OF THE SECOND DERIVATIVES OF THE HEAT TRANSFER RATE FOR
C     THE CROSS TERMS
C
      COMMON/KROS/D2QTWTS,D2QTWRW,D2QTWRS,D2QTWPT,D2QRWTS,D2QTSRS,D2QTSP
     1T,D2QRWRS,D2QRWPT,D2QRSPT
      D2QTWTS=GAM*((AD1*BETD1)-(ALPHD1*BEED1))/777.66
      D2QTWRW=GAWAD1*((A*BETD1)-(ALPHA*BEED1))/777.66
      D2QTWRS=GASD1*((A*BETD1)-(ALPHA*BEED1))/777.66
      D2QTWPT=GAPTD1*((A*BETD1)-(ALPHA*BEED1))/777.66
      D2QRWTS=GAWAD1*((AD1*BETA)-(ALPHD1*B))/(777.66)
      D2QTSRS=GASD1*((AD1*BETA)-(ALPHD1*B))/(777.66)
      D2QTSPT=GAPTD1*((AD1*BETA)-(ALPHD1*B))/(777.66)
      D2QRWRS=0.0136105*(DENS**(-0.85))*(PRE**0.25)*(DENWAL**(-0.9))*((B
     1ETA*A)-(ALPHA*B))/(777.66)
      D2QRWPT=0.0226841*(DENS**0.15)*(PRE**(-0.75))*(DENWAL**(-0.9))*((B
     1ETA*A)-(ALPHA*B))/(777.66)
      D2QRSPT=0.0340262*(DENS**(-0.85))*(PRE**(-0.75))*(DENWAL**0.1)*((B
     1ETA*A)-(ALPHA*B))/(777.66)
      RETURN
```

```
      END
************************************************************************
      SUBROUTINE NORM (QAVG,QTWD2,QTSD2,QROWD2,QROSD2,QPTD2,TW,TS,DENS,D
     1ENWAL,PRE)
C
C     NORMALIZES AND NON-DIMENSIONALIZES THE SECOND DERIVATIVES OF THE HEAT
C     TRANSFER RATE
C
      COMMON/KROS/D2QTWTS,D2QTWRW,D2QTWRS,D2QTWPT,D2QRWTS,D2QTSRS,D2QTSP
     1T,D2QRWRS,D2QRWPT,D2QRSPT
      WRITE (6,812)
  812 FORMAT(10X,"Q  NORMALIZED",/)
      QTWD2=QTWD2/QAVG
      QTSD2=QTSD2/QAVG
      QROWD2=QROWD2/QAVG
      QROSD2=QROSD2/QAVG
      QPTD2=QPTD2/QAVG
      D2QTWTS=D2QTWTS/QAVG
      D2QTWRW=D2QTWRW/QAVG
      D2QTWRS=D2QTWRS/QAVG
      D2QTWPT=D2QTWPT/QAVG
      D2QRWTS=D2QRWTS/QAVG
      D2QTSRS=D2QTSRS/QAVG
      D2QTSPT=D2QTSPT/QAVG
      D2QRWRS=D2QRWRS/QAVG
      D2QRWPT=D2QRWPT/QAVG
      D2QRSPT=D2QRSPT/QAVG
      QTWD2=QTWD2*(TW**2.0)
      QTSD2=QTSD2*(TS**2.0)
      QROWD2=QROWD2*(DENWAL**2.0)
      QROSD2=QROSD2*(DENS**2.0)
      QPTD2=QPTD2*(PRE**2.0)
      D2QTWTS=D2QTWTS*(TW*TS)
```

```
      D2QTWRW=D2CTWRW*(TW*DENWAL)
      D2QTWRS=D2CTWRS*(TW*DENS)
      D2QTWPT=D2CTWPT*(TW*PRE)
      D2QRWTS=D2CRWTS*(DENWAL*TS)
      D2QTSRS=D2CTSRS*(TS*DENS)
      D2QTSPT=D2CTSPT*(TS*PRE)
      D2QRWRS=D2CRWRS*(DENWAL*DENS)
      D2QRWPT=D2CRWPT*(DENWAL*PRE)
      D2QRSPT=D2CRSPT*(DENS*PRE)
      WRITE(6,819) QTWD2,QTSD2,QROWD2,QROSD2,QPTD2
  819 FORMAT(10X,"QTWD2=",E14.8/,10X,"QTSD2=",E14.8/,10X,"QROWD2=",E14.8
     1/,10X,"QROSD2=",E14.8/,10X,"QPTD2=",E14.8//)
      WRITE(6,821)D2QRWRS,D2QTSRS,D2QTWRS,D2QRSPT,D2QRWTS,D2QTWRW,D2QRWP
     1T,D2QTWTS,D2QTSPT,D2QTWPT
  821 FORMAT(10X,"D2QRWRS=",E14.6/,10X,"D2QTSRS=",E14.6/,10X,"D2QTWRS=",
     1E14.6/,10X,"D2QRSPT=",E14.6/,10X,"D2QRWTS=",E14.6/,10X,"D2QTWRW=",
     2E14.6/,10X,"D2QRWPT=",E14.6/,10X,"D2QTWTS=",E14.6/,10X,"D2QTSPT=",
     2E14.6/,10X,"D2QTWPT=",E14.6///)
      RETURN
      END

**************************************************************************

      SUBROUTINE FLUCTS(QAVG,QTWD2,QTSD2,QROWD2,QROSD2,QPTD2,QTWD1,QTSD1
     1,QROWD1,QROSD1,QPTD1,TW,TS,DENWAL,DENS,PRE,TSK,R)
C
C     CALCULATES THE HEAT TRANSFER RATE WITH FLUCTUATIONS AND COMPARES WITH
C     THE HEAT TRANSFER RATE WITHOUT FLUCTUATIONS...ALSO CHECKS THE ACCURACY
C     OF THE FULL TAYLOR SERIES EXPANSION
C
      COMMON/F/FFS(5,5),KK
      COMMON/KROS/D2QTWTS,D2QTWRW,D2QTWRS,D2QTWPT,D2QRWTS,D2QTSRS,D2QTSP
     1T,D2QRWRS,D2QRWPT,D2QRSPT
      COMMON/A/X(2),HFORM(2),ATOMS(2),THETA(2),ESPL(2,6),GL(2,6),VISCA,V
     1ISCB,XLEWA,XLEWB,XLEWC,XLEWD,HD(2),XMW(2),PR1,PR2,PR3,PR4,PR5,PR6,
```

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```
   2PR7,PR8,PR9,PR10,H1,H2,H3,H4,H5
    DO 60 L=1,5
    READ(5,311) (EPS(I,I),I=1,5)
311 FORMAT(5F10.3)
    READ(5,312) (EPS(1,I),I=2,5)
312 FORMAT(4F10.3)
    READ(5,313) (EPS(2,I),I=3,5)
313 FORMAT(3F10.3)
    READ(5,314) EPS(3,4),EPS(3,5),EPS(4,5)
314 FORMAT(3F10.3)
    WRITE(6,901) (EPS(I,I),I=1,5)
901 FORMAT(10X,"ERHOS=",F10.3/,10X,"ERHOW=",F10.3/,10X,"ETS=",F12.3/,1
   110X,"ETW=",F12.3/,10X,"EPT2=",F11.3)
    WRITE(6,302) (EPS(1,I),I=2,5)
302 FORMAT(10X,"ERWRS=",F10.3/,10X,"ETSRS=",F10.3/,10X,"ETWRS=",F10.3/
   1,10X,"ERSPT=",F10.3)
    WRITE(6,303) (EPS(2,I),I=3,5)
303 FORMAT(10X,"ERWTS=",F10.3/,10X,"ETWRW=",F10.3/,10X,"ERWPT=",F10.3)
    WRITE(6,304) EPS(3,4),EPS(3,5),EPS(4,5)
304 FORMAT(10X,"ETWTS=",F10.3/,10X,"ETSPT=",F10.3/,10X,"ETWPT=",F10.3/
   1///)
    QDOT1=1.0
    QDOT2=QDOT1+QROSD2*(EPS(1,1)**2.0)/2.0
    QDOT3=QDOT2+QROWD2*(EPS(2,2)**2.0)/2.0
    QDOT4=QDOT3+QTSD2*(EPS(3,3)**2.0)/2.0
    QDOT5=QDOT4+QTWD2*(EPS(4,4)**2.0)/2.0
    QDOT6=QDOT5+QPTD2*(EPS(5,5)**2.0)/2.0
    QDOT7=QDOT6+D2QRWRS*EPS(1,2)
    QDOT8=QDOT7+D2QTSRS*EPS(1,3)
    QDOT9=QDOT8+D2QTWRS*EPS(1,4)
    QDOT10=QDOT9+D2QRSPT*EPS(1,5)
    QDOT11=QDOT10+D2QRWTS*EPS(2,3)
    QDOT12=QDOT11+D2QTWRW*EPS(2,4)
    QDOT13=QDOT12+D2QRWPT*EPS(2,5)
    QDOT14=QDOT13+D2QTWTS*EPS(3,4)
```

```
      QDOT15=QDOT14+D2QTSPT*EPS(3,5)
      QDOT16=QDOT15+D2QTWPT*EPS(4,5)
      QDOTN=QDOT16
      QDOT=QDOTN*QAVG
      WRITE(6,618) QDOT,QDOTN
  618 FORMAT(10X,"QDOT*SQRT(R)-(BTU/FT1.5*SEC2)=",E16.8/,10X,"QDOT(NORMA
     1LIZED)=",E16.8///)
      QTOTA1=QDOT+QROSD1*EPS(1,1)*DENS
      QTOTA2=QTOTA1+QROWD1*EPS(2,2)*DENWAL
      QTOTA3=QTOTA2+QTSD1*EPS(3,3)*TS
      QTOTA4=QTOTA3+QTWD1*EPS(4,4)*TW
      QTOTAL=QTOTA4+QPTD1*EPS(5,5)*PRE
      DESF=DENS+EPS(1,1)*DENS
      DENWAF=DENWAL+EPS(2,2)*DENWAL
      TSF=TS+EPS(3,3)*TS
      TWF=TW+EPS(4,4)*TW
      PRF=PRE+EPS(5,5)*PRE
      CALL THERM(TWF,TSK,ENTHW)
      CALL THERM(TSF,TSK,ENTHS)
      CALL PRANUM (ENTHW,PR)
      CALL VISCTY (TWF,XMUW)
      CALL VISCTY (TSF,XMUS)
      CALL LEWIS (ENTHS,XLEWIS)
      CALL RATIO (ENTHS,HRATIO)
      QFLU=0.763*(PR**(-0.6))*((DENWAF*XMUW)**0.1)*((DESF*XMUS)**0.4)*(1
     1.0+(((XLEWIS**0.52)-1.0)*HRATIO))*(ENTHS-ENTHW)*(((2.0*PRF)/DESF)*
     2*0.25)*(1.0/777.66)
      DIFF=QTOTAL-QFLU
      PERCENT=DIFF/QTOTAL
      WRITE(6,619) DIFF,PERCENT,QFLU,QTOTAL
  619 FORMAT(10X,"DIFF=",E14.8/,10X,"PERCENT=",E14.8/,10X,"QFLU=",E14.8/
     1,10X,"QTOTAL=",E14.8/)
   60 CONTINUE
      RETURN
      END
```

APPENDIX C

RESULTS OF OTHER SPECIFIC CASES INVESTIGATED

APPENDIX C

Results of Other Specific Cases Investigated

Case 3

This case involved fixing the initial conditions and investigating the effects of the simultaneous fluctuations of the five independent variables. In this case the fluctuations are assumed to be uncorrelated. The five variables are fluctuated simultaneously over a range of $0 \leq \varepsilon \leq 1$. In other words,

$$\sqrt{\langle \varepsilon_1^2 \rangle} = \sqrt{\langle \varepsilon_2^2 \rangle} = \sqrt{\langle \varepsilon_3^2 \rangle} = \sqrt{\langle \varepsilon_4^2 \rangle} = \sqrt{\langle \varepsilon_5^2 \rangle}$$

and

$$\langle \varepsilon_i \varepsilon_j \rangle = 0$$

over the range of fluctuations mentioned.

The results of this investigation are shown in Figure 12. For all fluctuations considered, the heat transfer rate decreased and the magnitude of the percent error increased as the value of the fluctuations increased. The heat transfer rate decreased by -0.6 percent for the five independent variables fluctuated for $\varepsilon = 0.2$ and decreased by -15.8 percent for $\varepsilon = 1.0$. See Figure 12 for this case.

Case 4

This case also involved fixing the initial conditions and investigating the effects of the simultaneous fluctuations of the five independent variables. However, in this case, perfect positive correlation was considered

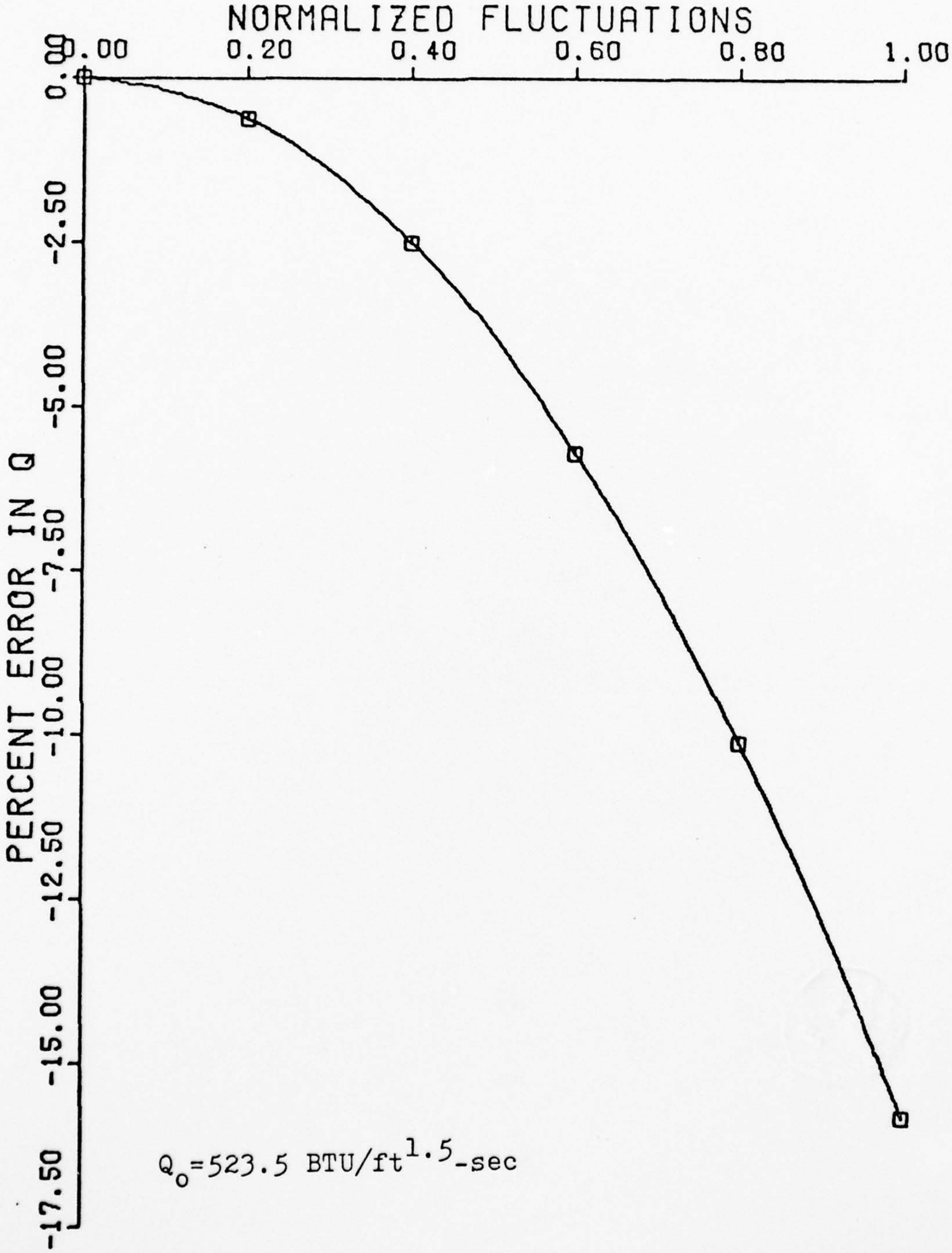

Fig 12. Percent Error in the Heat Transfer Rate for Fluctuations of all Variables - Uncorrelated (Baseline Initial Conditions)

$$\sqrt{\langle \varepsilon_1^2 \rangle} = \sqrt{\langle \varepsilon_2^2 \rangle} = \sqrt{\langle \varepsilon_3^2 \rangle} = \sqrt{\langle \varepsilon_4^2 \rangle} = \sqrt{\langle \varepsilon_5^2 \rangle}$$

and

$$\langle \varepsilon_i \varepsilon_j \rangle = \sqrt{\langle \varepsilon_i^2 \rangle} \sqrt{\langle \varepsilon_j^2 \rangle}$$

The range of fluctuations was again $0 \leq \varepsilon \leq 1$. The results of this investigation are shown in Figure 13. The addition of the cross terms signficantly increased the value of the heat transfer rate. The heat transfer rate was increased by 2.5 percent when $\varepsilon = 0.2$ and was increased up to 63.4 percent for $\varepsilon = 1.0$. This was a significant increase in the heat transfer rate; therefore, the effect of the cross terms is investigated further in the next case.

Case 5

This case involved fixing the initial conditions and choosing two of the independent variables to be fluctuated. Positive correlation was assumed for each set of two variables chosen. There were many combinations of two variables that could be chosen. The magnitudes of the second derivatives of the heat transfer rate used in the Taylor Series expansion were observed to determine which terms appeared to have greater effect on the heat transfer rate. The magnitudes of these non-dimensionalized, normalized derivatives for the baseline conditions were

$$\frac{\partial^2 Q}{\partial \rho_s^2} = -0.1275 \qquad \frac{\partial^2 Q}{\partial \rho_s \partial T_s} = 0.0375$$

$$\frac{\partial^2 Q}{\partial \rho_w^2} = -0.0900 \qquad \frac{\partial^2 Q}{\partial \rho_w \partial T_s} = 0.1446$$

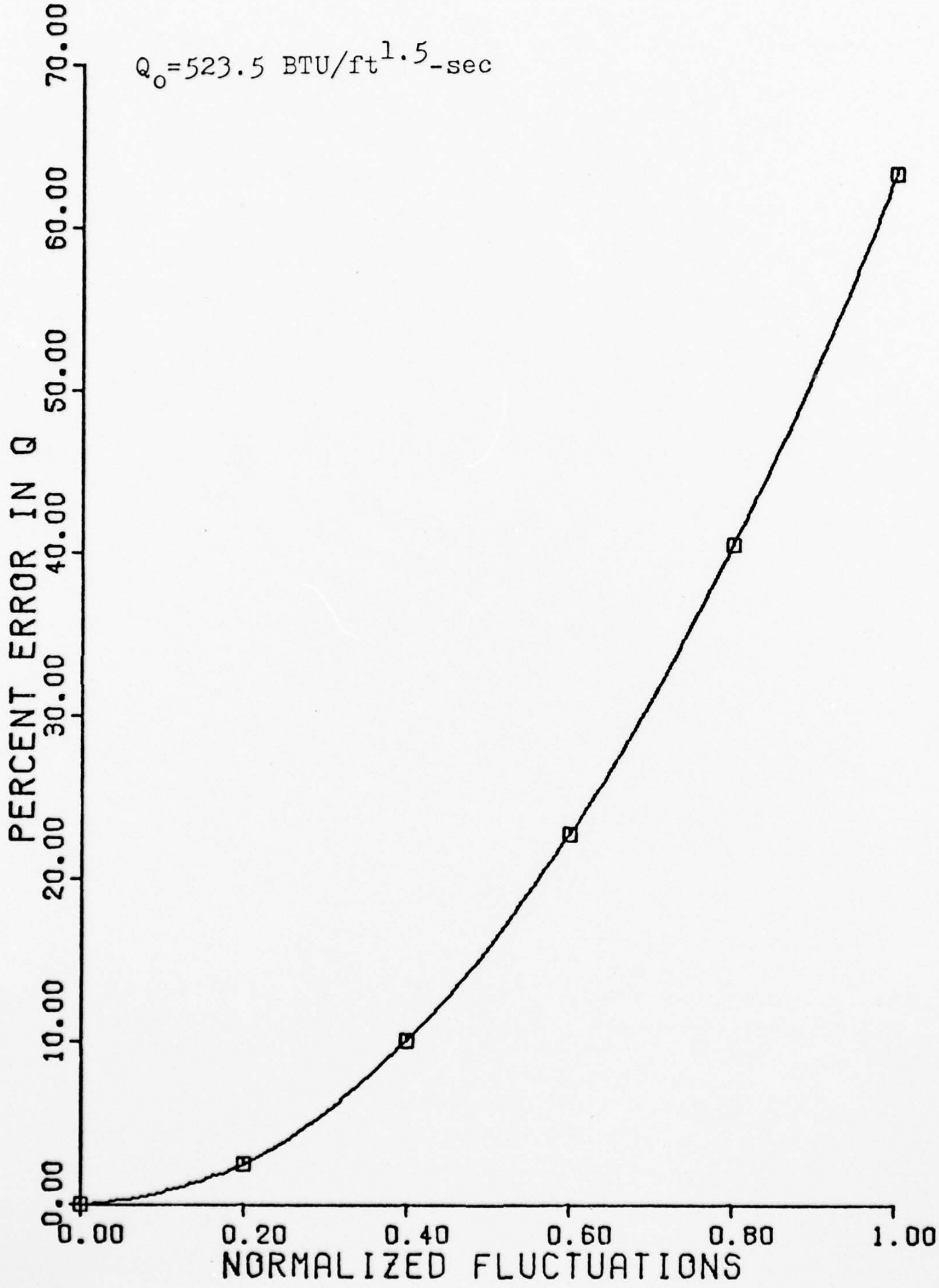

Fig. 13. Percent Error in the Heat Transfer Rate for Fluctuations of all Variables - Correlated (Baseline Initial Conditions)

$$\frac{\partial^2 Q}{\partial T_s^2} = 0.3483 \qquad \frac{\partial^2 Q}{\partial \rho_w \partial T_w} = -0.0097$$

$$\frac{\partial^2 Q}{\partial T_w^2} = -0.2589 \qquad \frac{\partial^2 Q}{\partial \rho_w \partial P_{t_2}} = 0.0250$$

$$\frac{\partial^2 Q}{\partial P_{t_2}^2} = -0.1875 \qquad \frac{\partial^2 Q}{\partial T_w \partial T_s} = 0.0397$$

$$\frac{\partial^2 Q}{\partial \rho_s \partial \rho_w} = 0.0150 \qquad \frac{\partial^2 Q}{\partial T_s \partial P_{t_2}} = 0.3615$$

$$\frac{\partial^2 Q}{\partial T_s \partial \rho_s} = 0.2169 \qquad \frac{\partial^2 Q}{\partial T_w \partial P_{t_2}} = -0.0244$$

$$\frac{\partial^2 Q}{\partial \rho_s \partial T_w} = -0.1463$$

The absolute value of the magnitudes of four of the cross term derivatives was greater than 0.1. These derivatives are $\frac{\partial^2 Q}{\partial T_s \partial \rho_s}$, $\frac{\partial^2 Q}{\partial \rho_s \partial T_w}$, $\frac{\partial^2 Q}{\partial \rho_w \partial T_s}$, and $\frac{\partial^2 Q}{\partial T_s \partial P_{t_2}}$. Since three of these derivatives involved a fluctuation in the temperature at the edge of the boundary layer, all four cross terms involving this variable were investigated. Additionally, an investigation was made of the $\frac{\partial^2 Q}{\partial \rho_s \partial T_w}$ term. For each case checked, the two variables were fluctuated simultaneously over a range of $0 \leq \varepsilon \leq 1$.

$$\sqrt{\langle \varepsilon_i^2 \rangle} = \sqrt{\langle \varepsilon_j^2 \rangle}$$

$$\langle \varepsilon_i \varepsilon_j \rangle = \sqrt{\langle \varepsilon_i^2 \rangle} \sqrt{\langle \varepsilon_j^2 \rangle}$$

All of the cases involving fluctuations of the temperature at the edge of the boundary layer increased the heat transfer rate. When it was fluctuated with the density at the edge of the boundary layer, the heat transfer rate was

increased by 1.2 percent for $\varepsilon = 0.2$ and up to 32.7 percent for $\varepsilon = 1.0$. Similar results were obtained when T_s was fluctuated with the wall density. The results of these two cases are shown in Figure 14.

The effect on the heat transfer rate was the largest when the temperature at the edge of the boundary layer was fluctuated simultaneously with the stagnation pressure. The heat transfer rate was increased by 1.7 percent for $\varepsilon = 0.2$ and up to 44.2 percent for $\varepsilon = 1.0$. There was a much smaller effect when the two temperature variables were fluctuated together. The results of these two cases are shown in Figure 15.

Simultaneous fluctuations of the wall temperature and the density at the edge of the boundary layer revealed a significant decrease in the heat transfer rate. It was decreased by -0.9 percent for $\varepsilon = 0.2$ and by -20.8 percent for $\varepsilon = 1.0$. Results of this case are shown in Figure 16.

In addition to investigating these five cases, some interesting results were obtained from investigating the analytical expressions of the various second derivatives in the Taylor Series expansion. Inspection of the cross terms involving combinations of the two densities and the stagnation pressure revealed that the magnitude of these second derivatives could be easily obtained. The three second derivatives are simply the product of a constant and the heat transfer rate without fluctuations. Although those terms have a small effect compared to some of the other

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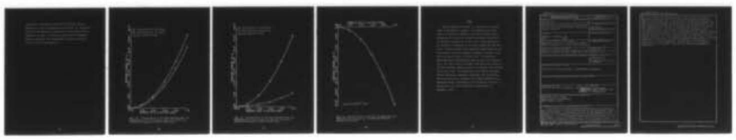

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cross terms, inclusion of them into the Taylor Series expansion is easily accomplished if desired. An inspection of any of the analytical derivatives in the Taylor Series expansion can lead to conclusions about their dependency upon the transport and thermodynamic properties as well as the other flow parameters.

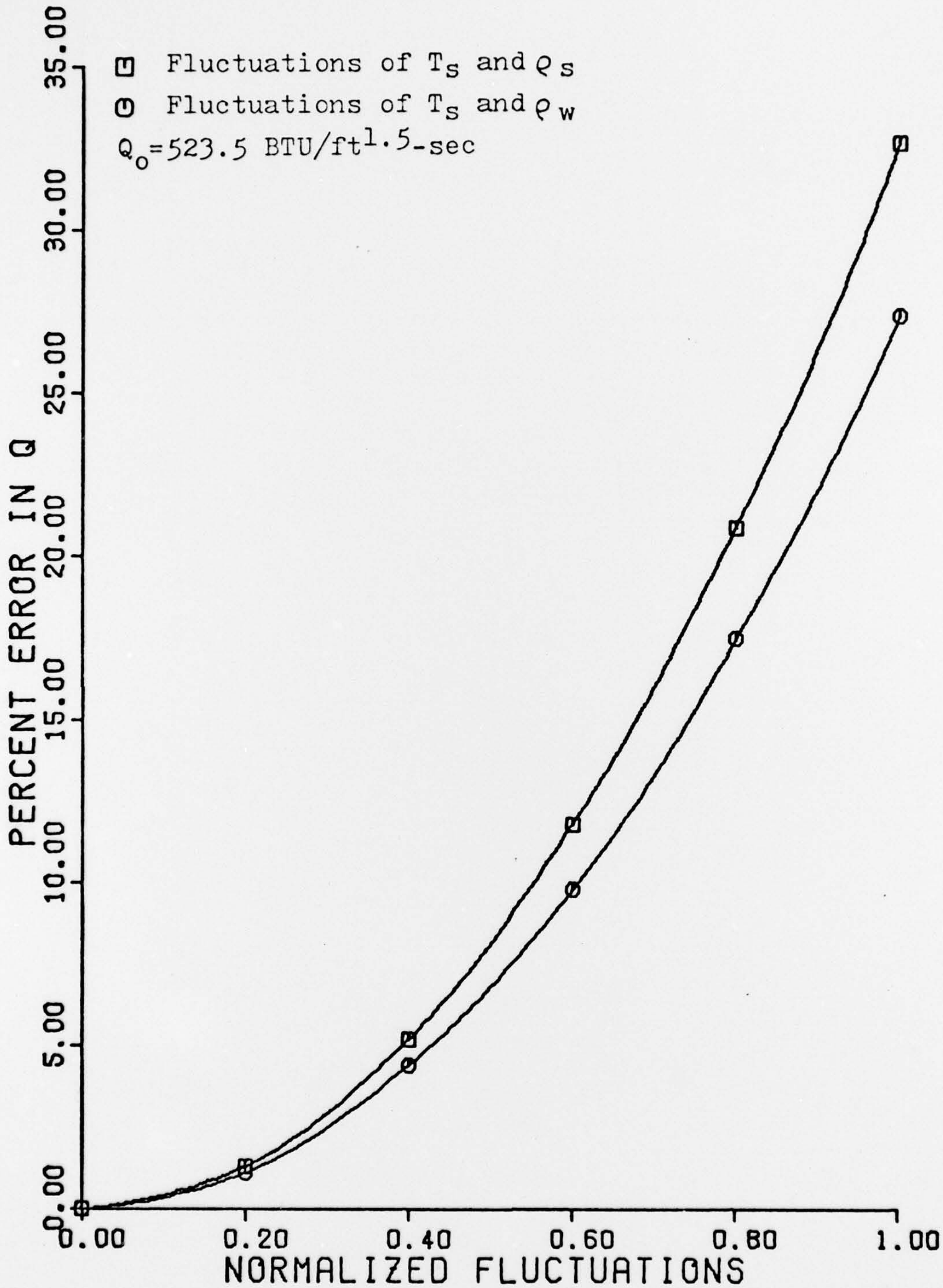

Fig. 14. Percent Error in the Heat Transfer Rate for Fluctuations of (a) T_s and ρ_s, and (b) T_s and ρ_w - Correlated (Baseline Initial Conditions)

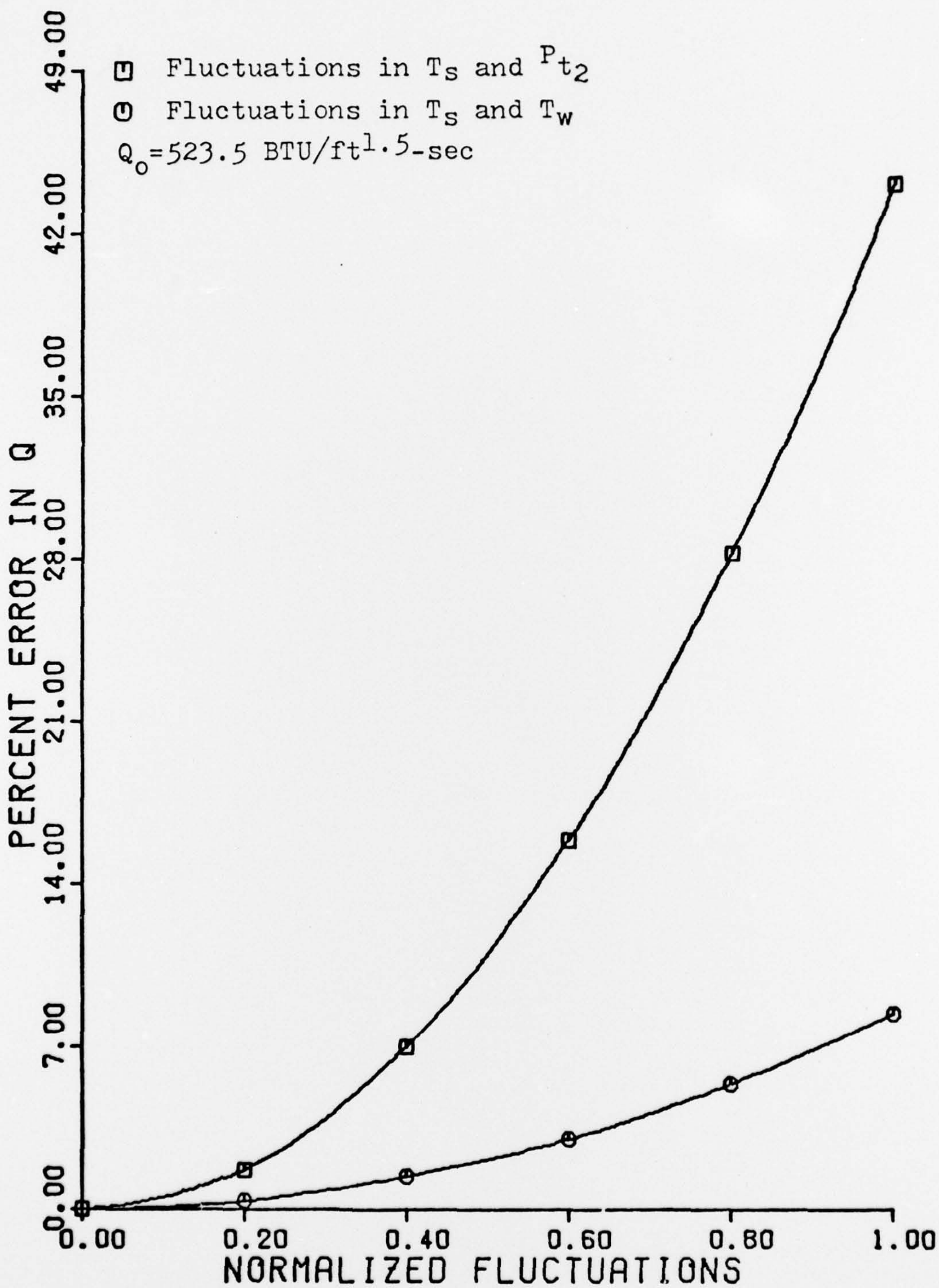

Fig. 15. Percent Error in the Heat Transfer Rate for Fluctuations of (a) T_s and P_{t_2}, and (b) T_s and T_w - Correlated (Baseline Initial Conditions)

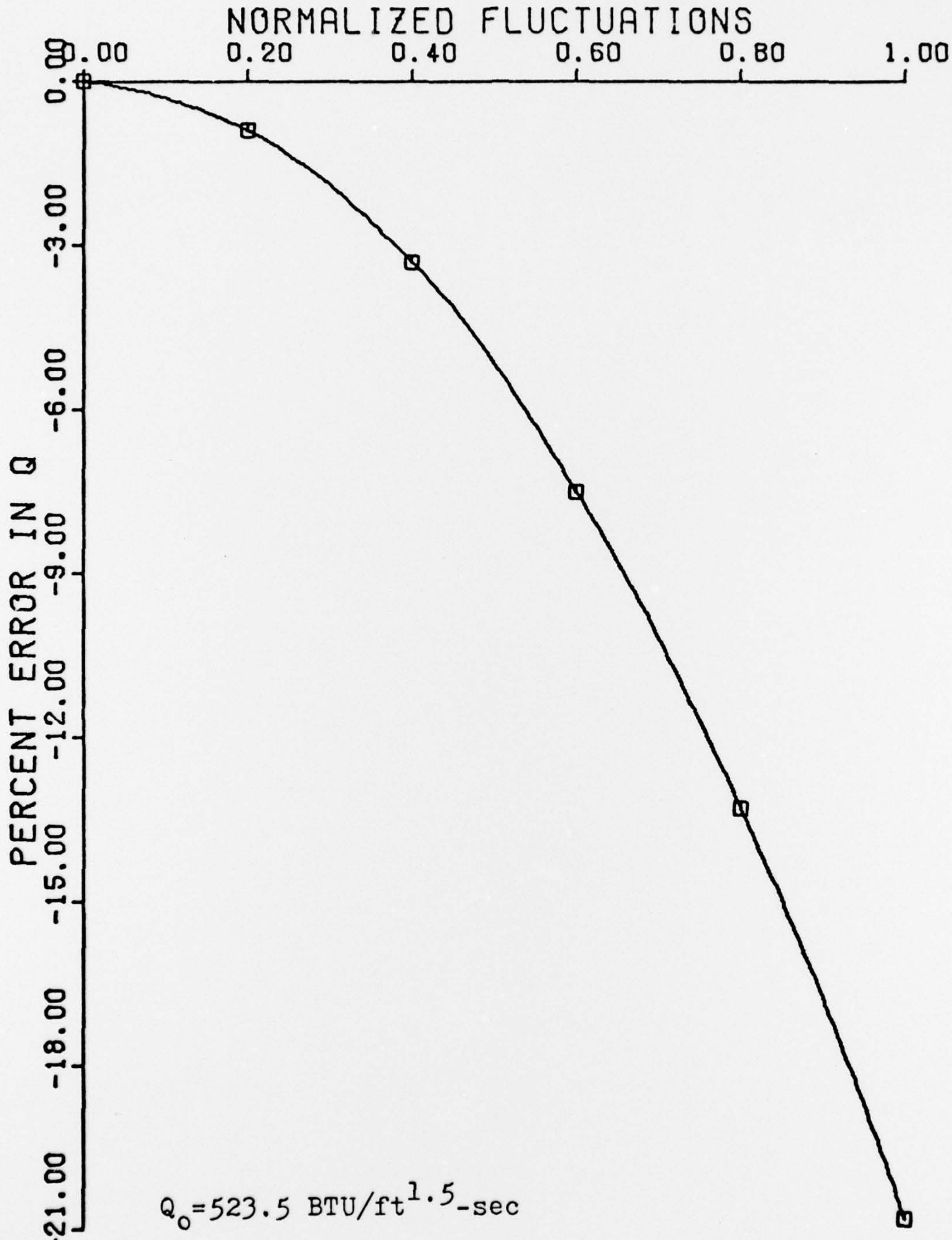

Fig. 16. Percent Error in the Heat Transfer Rate for Fluctuations of ρ_s and T_w - Correlated (Baseline Initial Conditions)

Vita

William Humphrey Barnett, Jr. was born on 13 March, 1948, in Montgomery, Alabama. He graduated from high school in Opelika, Alabama in 1966, and attended Auburn University from which he received the degree of Bachelor of Aerospace Engineering in May, 1971. Upon graduation, he received a commission in the USAF through the ROTC program. He was employed as an associate design engineer for the Southern Services Company, Birmingham, Alabama, until called on active duty in October 1971. He completed Minuteman Missile Maintenance Training School in January, 1972. He then served as a Combat Targeting Team Officer in the 90th Strategic Missile Wing, F. E. Warren, AFB, Wyoming, until July, 1974. He transfered to the 390lst Strategic Missile Evaluation Squadron, Vandenberg AFB, California, where he served as a Strategic Air Command Evaluator for Minuteman Missile Maintenance until entering the School of Engineering, Air Force Institute of Technology, in September, 1976.

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES
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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The effects of flow property fluctuations on the stagnation point heat transfer rate, as calculated by the Fay-Riddell equation, are investigated. Fluctuations in free stream temperature and density, wall temperature and density, and stagnation pressure are considered. The flow parameter perturbations are included in the Fay-Riddell equation by approximating the heat transfer rate by a Taylor Series truncated to second order and expanded about the mean values of the five independent variables. Taking the time averaged

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next page

of this expression, the first order terms in the fluctuations drop out, and an expression for the time averaged stagnation point heat transfer rate is obtained. A comparison between the laminar steady flow heat transfer rate and the heat transfer rate with fluctuations shows that neglecting the fluctuations in the five independent variables can lead to errors in the calculated heat transfer rate. Of the five variables considered, the fluctuations of the temperature at the edge of the boundary layer have the greatest effect on the heat transfer rate. Fluctuations of half the initial value of this variable result in a 5 to 10 percent change in the heat transfer rate over the range of initial conditions considered. The error increases significantly with the magnitude of the fluctuations. For example, a 100 percent fluctuation in this temperature results in a 15 to 35 percent error in the heat transfer rate for the range of temperature considered.